**APPM 1360** **EXAM #2** **SPRING 2001**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name, (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (25 points) Evaluate the following:

   (a) $\lim_{n\to\infty} n^n e^{-2n}$

   (b) $\lim_{n\to\infty} \cos(n\pi)\left(\frac{1}{n}\right)^n$

   (c) $3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

   (d) $\sum_{n=1}^{\infty} \tan^{-1}(n-1) - \tan^{-1}(n)$

   (e) $\int_{-1}^{\infty} \frac{dx}{x^2+5x+6}$

2. (30 points) Determine whether the following items converge or diverge, and state clearly the test used.

   (a) $\int_0^{2a} \frac{dx}{(x-a)^2}$

   (b) $\int_1^{\infty} \frac{1}{x^3}\sqrt{1-\frac{1}{x^6}}\,dx$

   (c) $\sum_{n=1}^{\infty} ne^{-n}$

   (d) $\sum_{n=3}^{\infty} \frac{\ln n}{n}$

   (e) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

   (f) $\sum_{n=1}^{\infty} \frac{2^{(n^2)}}{n^n}$

3. (20 points) For each of the following statements, clearly write either TRUE or FALSE in your bluebook. There is no parital credit, and credit will be given only for the correct BOXED answer.

   (a) If $\sum_{n=0}^{\infty} |a_n|$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

   (b) If $\sum_{n=0}^{\infty} |a_n|$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

   (c) If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} |a_n|$ converges.

   (d) If $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

   (e) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

   (f) If $\lim_{n\to\infty} a_n = 1$, then $\sum_{n=0}^{\infty} a_n$ diverges.

   (g) The ratio test can be used to show that $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges.

   (h) The ratio test can be used to show that $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

   (i) $\int_{-1}^{1} \frac{dx}{x^3} = 0$.

   (j) If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges.

COOL, THERE'S MORE — TURN THE PAGE OVER!

4. (25 points) Consider the infinite series $\sum_{n=0}^{\infty}(-x^2)^n$. If $|x^2| < 1$, it converges to the limit $\frac{1}{1+x^2}$, which is a function that we will call $f(x)$.

(a) Write out the first **four** terms of the series.

(b) If the first **two** terms of the series are used to approximate $f(x) = \frac{1}{1+x^2}$, give an upper estimate for the magnitude of the resulting error.

(c) Will your approximation from part (b) result in a value that is larger, or smaller, than the true value of $f(x) = \frac{1}{1+x^2}$? (Be sure to state the reason for your answer.)

(d) Now, suppose that $x = 0.1$. Use this specific value of $x$ to verify your results from parts (b) and (c). Be sure to show all of your work. (Hint: $\frac{100}{101} = 0.9900\overline{9900}$.)

**APPM 1360** **Exam #2 Formula Sheet** **Spring, 2001**

1. A short table of integrals. In the following, $a \neq 0$.

(a) $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

(b) $\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

(c) $\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

(d) $\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

(e) $\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

(f) $\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

(g) $\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

(h) $\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. In formulas (3)–(6), $x$ remains fixed as $n \to \infty$.

(a) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

(b) $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(c) $\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$

(d) $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$

(e) $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad$ (Any $x$)

(f) $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad$ (Any $x$)