

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name, (4) a grading table. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (20 points) For each of the following functions, determine the Maclaurin series and the corresponding radius of convergence.

(a)  $f(x) = \frac{1}{1+2x}$

(c)  $f(x) = \frac{x^3}{(1-x)^2}$

(b)  $f(x) = \frac{\sin x}{x}$

(d)  $f(x) = \int_0^x e^{-t^2} dt$

2. (25 points) Consider the power series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n 2^n}$ .

- (a) For what values of  $x$  does the power series converge absolutely?  
 (b) For what values of  $x$  does the power series converge conditionally?  
 (c) What is the interval of convergence?

3. (25 points) Consider using the second order Taylor polynomial  $P_2(x)$  approximation for  $\cos(x)$  to estimate the value of  $\cos(1.1\pi)$ .

- (a) If the center  $a = 0$  is used, write out  $P_2(x)$ .  
 (b) Estimate the error when  $P_2(x)$  is used to approximate  $\cos(1.1\pi)$ . (Leave your answer in terms of factorials, etc.)  
 (c) Now, assume you use a new center  $a = \frac{\pi}{2}$ . Write out  $P_2(x)$ .  
 (d) Estimate the error when  $P_2(x)$  from part (c) is used to approximate  $\cos(1.1\pi)$ .

4. (20 points) Consider the function  $\sinh x = (e^x - e^{-x})/2$ , which may be represented by the power series  $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$  which is known to converge for all  $x$ .

- (a) Calculate the power series for  $\cosh x$  by differentiating, term-by-term, the power series for  $\sinh x$ .  
 (b) Now, verify your result in part (a) by integrating the power series for  $\sinh x$ .  
 (c) On what interval does your power series for  $\cosh x$  converge? Be sure to justify your answer!

5. (10 points) Classify the following curve and determine the eccentricity and the coordinates of the center, foci and vertices.

$$4(x+2)^2 + 9(y-1)^2 = 36$$