

On front of your bluebook, write your NAME and a GRADING TABLE.

SHOW YOUR WORK: Correct answers with no work shown will receive no credit! Incorrect answers with correct work shown will receive partial credit. *When determining convergence or divergence, state clearly what test you are using.*

1. (25 points) Evaluate the following integrals. If an integral is improper, determine whether it converges or diverges. (If it converges, evaluate it!)

$$(a) \int \frac{4v^4 + 19v^2 + 4v + 4}{2v^3 + 8v} dv \quad (b) \int \frac{1}{y^2 \sqrt{9 - y^2}} dy \quad (0 < y < 3)$$

$$(c) \int_0^1 (x + 1)^2 e^x dx \quad (d) \int_0^3 \frac{1}{(x - 2)^2} dx$$

2. (25 points) Determine whether the following integrals converge or diverge. **You do not need to evaluate them.**

$$(a) \int_0^{\infty} \frac{dx}{e^x + e^{-x}} \quad (b) \int_5^{\infty} \frac{s + 1}{\sqrt{s^4 + 3s}} ds$$

3. (25 points) Find out whether the following series converge or diverge. **If a series converges, find its sum.**

$$(a) \sum_{n=3}^{\infty} \left(1 - \frac{1}{2n}\right)^n \quad (b) \sum_{n=1}^{\infty} \left(\frac{2}{2n+1} - \frac{2}{2n+3}\right) \quad (c) \sum_{n=1}^{\infty} e^{-n}$$

4. (25 points) Determine whether the following series converge or diverge.

$$(a) \sum_{n=1}^{\infty} \frac{8^{n+1} n!}{(n+2)! 7^n} \quad (b) \sum_{n=2}^{\infty} \frac{n}{(\ln n)^{\frac{n}{2}}} \quad (c) \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{\frac{5}{2}}}$$

5. **EXTRA CREDIT** (5 points) Find out whether the following statement is true or false. If it is true, justify why. If it is false, find a counterexample:

$$\text{If } \sum_{n=1}^{\infty} a_n \text{ diverges, then } \sum_{n=1}^{\infty} \frac{a_n}{\sqrt{n}} \text{ diverges.}$$

♣ GOOD LUCK! ♣

1. A short table of integrals. In the following, $a \neq 0$.

$$(a) \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C \text{ for } u^2 < a^2$$

$$(b) \int \frac{du}{a^2 + u^2} = (1/a) \tan^{-1}(u/a) + C$$

$$(c) \int \frac{du}{u\sqrt{u^2 - a^2}} = (1/a) \sec^{-1} |u/a| + C \text{ for } u^2 > a^2 + C$$

$$(d) \int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}(u/a) + C \text{ for } a > 0$$

$$(e) \int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}(u/a) + C \text{ for } u > a > 0$$

$$(f) \int \frac{du}{a^2 - u^2} = \begin{cases} (1/a) \tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a) \coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$$

$$(g) \int \frac{du}{u\sqrt{a^2 - u^2}} = -(1/a) \operatorname{sech}^{-1}(u/a) + C \text{ for } 0 < u < a$$

$$(h) \int \frac{du}{u\sqrt{a^2 + u^2}} = -(1/a) \operatorname{csch}^{-1} |u/a| + C \text{ for } u \neq 0$$

2. Some identities.

$$(a) \sin^2 x + \cos^2 x = 1$$

$$(b) \sin^2 x = (1 - \cos(2x))/2$$

$$(c) \cos^2 x = (1 + \cos(2x))/2$$

$$(d) \cosh^2 x - \sinh^2 x = 1$$

3. Some useful trig substitutions.

$$(a) x = a \tan \theta \text{ replaces } a^2 + x^2 \text{ by } a^2 \sec^2 \theta$$

$$(b) x = a \sin \theta \text{ replaces } a^2 - x^2 \text{ by } a^2 \cos^2 \theta$$

$$(c) x = a \sec \theta \text{ replaces } x^2 - a^2 \text{ by } a^2 \tan^2 \theta$$

4. Some useful limits.

$$(a) \lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$$

$$(b) \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$(c) \lim_{n \rightarrow \infty} x^{1/n} = 1 \text{ for } x > 0$$

$$(d) \lim_{n \rightarrow \infty} x^n = 0 \text{ for } |x| < 1$$

$$(e) \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x \text{ for any } x$$

$$(f) \lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0 \text{ for any } x$$

$$(g) \lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0$$

$$(h) \lim_{z \rightarrow 0} \frac{\sin z}{z} = 1$$

Note: in (c) - (f), x remains **fixed** as $n \rightarrow \infty$.