APPM 1360 **Exam #3** Nov 28, 2001

On the front of your bluebook, print your name, the name of your instructor, when your class meets, & Exam #3.

There are 4 questions. Answer all parts of all 4 questions. Show all your work in your bluebook (a correct answer with no work may receive no credit). Start each question on a new page. Box in your answers. **No** calculators are allowed. You may have one handwritten sheet of formulae.

1. (25 points) Which of the following series converge absolutely, which converge conditionally, and which diverge? Justify your answer.

a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\pi}$ b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ c) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n^2}$

2. (25 points) For what values of x do the following series converge (*i*) absolutely, (*ii*) conditionally? Justify your answer.

a) $\sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^{2n}}{n4^n}$ b) $\sum_{n=0}^{\infty} (\ln x)^n$ c) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$

3. (25 points)

a) Use series to evaluate the limit: $\lim_{x \to 2} \frac{x^2 - 4}{\ln(x-1)}$.

b) Use series to evaluate the integral: $f(x) = \int_0^x t^2 e^{-t}\, dt$.

c) Find $f^{(5)}(0)$ when $f(x) = \frac{1}{x}\ln(1+x^2)$ ($f(x)$ is defined to be zero at $x = 0$).

4. (25 points) Let $f(x) = e^{x^2}$

a) Find the Maclaurin series for $f(x)$, and explicitly give the Maclaurin polynomial P_2.

b) Give an exact expression for the Taylor remainder $R_2(x)$.

c) Using (b) estimate the error in the approximation $e \approx P_2(1)$.

Extra Credit: (10 points) Does the series

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{\sqrt{n} - (-1)^n}$$

converge absolutely, converge conditionally or diverge? Explain.