APPM 1360 — Special Final Exam — December 9, 2002

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You are respnsible for all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Only the provided formula sheet is permitted (no textbooks, classnotes, crib sheets, or calculators). $\boxed{\text{Box}}$ in your answers, if possible. **Please start each new problem on a new page.**

1. (18 points) Find dy/dx for each of the following:

a) $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

b) $x(t) = 4\sin(t) + \sin(5)$,
$y(t) = \sqrt{3}\cos(t)$

2. (20 points) Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage V across its terminals and that, if t is measured in seconds, $\dfrac{dV}{dt} = -\dfrac{1}{40}V$. Solve this equation for V, using V_0 to denote the value of V when $t = 0$. How long will it take the voltage to drop to 10% of its original value?

3. (16 points) Evaluate each of the following integrals.

a) $\displaystyle\int \frac{1}{x^2+6x+25}\,dx$

b) $\displaystyle\int \frac{1}{x^2+6x+8}\,dx$

4. (16 points) Explain whether each of the following integrals converge or diverge.

a) $\displaystyle\int_1^\infty \frac{x^2+1}{x^5+x+1}\,dx$

b) $\displaystyle\int_0^2 \frac{1}{1-x}\,dx$

5. (30 points) True or False. If the statement is true, write the word TRUE and provide a short explanation of why it is true. If it is false, write the word FALSE and give an example showing that it is false.

a) If $\lim\limits_{n\to\infty} a_n = 0$ then $\displaystyle\sum_{n=1}^\infty a_n$ converges.

b) If $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = 1$ then $\displaystyle\sum_{n=1}^\infty a_n$ diverges.

c) If $f(x)$ is a differentiable function such that $f(0) = 1$, $f(5) = 2$ and $\int_0^5 f(x)dx = 3$ then $\displaystyle\int_0^5 xf'(x)\,dx = 7$

d) The series $\displaystyle\sum_{n=1}^\infty n(1+n^2)^p$ converges if $p < -1$.

e) The Taylor series for e^x centered at $a = 1$ is $\displaystyle\sum_{k=0}^\infty \frac{(x-1)^(2k)}{(2k)!}$.

6. (15 points) Determine whether the following series converge absolutely, conditionally, or diverge. Indicate clearly **WHY** your answer is correct.

a) $\sum_{n=1}^{\infty}(-1)^n \ln(n)$

b) $\sum_{n=1}^{\infty} \frac{3\ln(n)+2}{n^2+2n+1}$

c) $\sum_{n=0}^{\infty} \frac{n^n}{3^n}$

7. (15 points) For what values of x do the following series converge (i) absolutely, (ii) conditionally? Justify your answers.

a) $\sum_{n=1}^{\infty} \frac{n^5(x-3)^{2n}}{4^n}$

b) $\sum_{n=1}^{\infty} \frac{n(\ln x)^{n-1}}{x}$

c) $\sum_{n=1}^{\infty} \frac{2^n x^n}{(3n)!n!}$

8. (30 points) Let $r_1 = \cos\theta$ and $r_2 = 1 - \cos\theta$ be two equations given in polar coordinates.

a) Calculate the length of the perimeter of $r_1 = \cos\theta$.

b) Find all the intersection points between r_1 and r_2.

c) Sketch the curves described by r_1 and r_2. Label the intersection points on your graph.

d) Find the area inside the graph of r_1 and outside the graph of r_2.

9. (20 points) Determine $P_3(x)$, the Taylor polynomial of order 3 generated by $f(x) = x^5 + x + 1$ at $x = 1$. Give $R_3(x)$, the error term when $P_3(x)$ is used to approximate $f(x)$.

10. (20 points) Let $f(n) = \int_0^{\infty} t^{n-1}e^{-t}\,dt$ for all positive integers n.

a) Show that $f(1) = 1$.

b) Use integration by parts to show that $f(n+1) = nf(n)$.

c) Use part (b) to give a simple formula for $f(n)$, for any integer $n \geq 1$.