APPM 1360 EXAM #1 Fall 2003

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (20 points) Compute the derivatives of the following functions and simplify your answers.
   (a) $y = \tanh^{-1}(\sin x)$
   (b) $y = x\tan^{-1}(-x) + \ln\left(\sqrt{1+x^2}\right)$
   (c) $y = x^x$
2. (20 points) Evaluate the following integrals and simplify your answers.
   (a) $\int_0^{\pi/4} e^{\sin(2x)}\cos(2x)\,dx$
   (b) $\int \frac{dx}{\sqrt{x^2+6x+18}}$
   (c) $\int \frac{e^x}{1+e^{2x}}\,dx$
3. (20 points) Consider the region in the first quadrant bounded above by the curve $y = \cosh(x)$ and on the right by the line $x = 1$.
   (a) Calculate the arc length of the curve bounding the upper portion of the region.
   (b) Calculate the surface area generated by revolving the upper bounding curve about the x-axis.
   (c) Set up, *but do not evaluate*, a single definite integral to calculate the volume generated by revolving the region about the y-axis.
   (d) Set up, *but do not evaluate*, a single definite integral to calculate the volume generated by revolving the region about the horizontal line $y = 3$.
4. (20 points) Consider a flat plate with uniform density ρ (mass per unit volume) and uniform thickness t whose shape is the region in the first quadrant bounded above by $y = \cos x$ and below by $y = \sin x$, for $0 \le x \le \frac{\pi}{4}$.
   (a) Calculate the total mass of the plate, m_{total}.
   (b) Calculate the moment of the region about the x-axis, M_x.
   (c) Determine the y coordinate of the center of mass of the plate, $\bar{y}$.
5. (20 points) Solve the following initial value problem for $y(x)$:

$$\begin{aligned} \cos(\ln y)\frac{dy}{dx} &= y \\ y(0) &= e^{\pi/2}. \end{aligned}$$

APPM 1360 **Exam #1 Formula Sheet** **SPRING 2003**

A short table of integrals. In the following, $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \quad \text{for } a > 0$

2. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad \text{for } u^2 < a^2$

3. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \quad \text{for } u > a > 0$

4. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

5. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$

6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \quad \text{for } u \neq 0$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \quad \text{for } 0 < u < a$

8. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \quad \text{for } u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$

2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$

3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

4. $\cosh^2 x - \sinh^2 x = 1$

5. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$

6. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$