

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (40 points) Compute the following integrals. Be sure to show all of your work to receive full credit.

$$\begin{array}{ll} \text{(a)} \int x \ln x \, dx & \text{(c)} \int_0^{1/2} \frac{4}{(4x^2 + 1)^2} \, dx \quad (\text{Hint: think trig}) \\ \text{(b)} \int \frac{3x + 2}{x^2 - 2x + 1} \, dx & \text{(d)} \int_0^\infty \frac{dx}{e^x} \end{array}$$

2. (20 points) Without evaluation, determine whether the following integrals converge or diverge. Be sure to justify your answers.

$$\begin{array}{ll} \text{(a)} \int_1^\infty \frac{dx}{e^x - 2^x} & \text{(b)} \int_2^\infty \frac{dx}{\ln x} \end{array}$$

3. (20 points) Determine whether the following sequences converge or diverge. If the sequence converges, find the value. Be sure to show all of your work and justify your answer.

$$\begin{array}{ll} \text{(a)} a_n = \frac{n + n(-1)^n}{n!} & \text{(b)} a_n = (n^2 + n)^{1/n} \end{array}$$

4. (20 points) Determine whether the following series converge or diverge. If the series converges, find the sum. Be sure to show all of your work and justify your answer.

$$\begin{array}{ll} \text{(a)} \sum_{n=2}^\infty \frac{\ln n}{n} & \text{(c)} \sum_{n=1}^\infty \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \\ \text{(b)} \sum_{n=0}^\infty \frac{2^n - 1}{3^n} & \end{array}$$

A short table of integrals. In the following, $a \neq 0$.

1. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1} \left(\frac{u}{a} \right) + C \quad \text{for } a > 0$
2. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \left(\frac{u}{a} \right) + C \quad \text{for } u^2 < a^2$
3. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \left(\frac{u}{a} \right) + C \quad \text{for } u > a > 0$
4. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + C$
5. $\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \left(\frac{u}{a} \right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a} \coth^{-1} \left(\frac{u}{a} \right) + C & \text{if } u^2 > a^2 \end{cases}$
6. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left| \frac{u}{a} \right| + C \quad \text{for } u \neq 0$
7. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{u}{a} \right) + C \quad \text{for } 0 < u < a$
8. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \left| \frac{u}{a} \right| + C \quad \text{for } u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \frac{1 + \cos(2x)}{2}$
3. $\sin^2 x = \frac{1 - \cos(2x)}{2}$
4. $\cosh^2 x - \sinh^2 x = 1$
5. $\cosh^2 x = \frac{\cosh(2x) + 1}{2}$
6. $\sinh^2 x = \frac{\cosh(2x) - 1}{2}$

In formulas (3)–(6), x remains fixed as $n \rightarrow \infty$.

1. $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$
2. $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$
3. $\lim_{n \rightarrow \infty} x^{1/n} = 1 \quad (x > 0)$
4. $\lim_{n \rightarrow \infty} x^n = 0 \quad (|x| < 1)$
5. $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n} \right)^n = e^x \quad (\text{Any } x)$
6. $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0 \quad (\text{Any } x)$