

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (30 points) Determine whether the following series converge or diverge. Give specific reasons for your answers.

(a)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}$

(d)  $\sum_{n=1}^{\infty} \frac{1 + \sin(n)}{n^3}$

(b)  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

(e)  $\sum_{n=1}^{\infty} \frac{3^n n^3}{n!}$

(c)  $\sum_{n=0}^{\infty} \frac{3n^2 + 5n + 4}{\sqrt{n^6 + 5n^3 + 4}}$

2. (25 points) Consider the power series  $\sum_{n=0}^{\infty} \frac{(-1)^n (x-4)^n}{n 2^n}$ .

- (a) For what values of  $x$  does the power series converge absolutely?  
(b) For what values of  $x$  does the power series converge conditionally?  
(c) What is the interval of convergence?
3. (20 points) We would like to use the function  $f(x) = e^{2x}$  to estimate the value of  $e^2$ . To do this we will use the third-order Taylor polynomial  $P_3(x)$  with a center of  $a = 0$ , evaluated at  $x = 1$ . Estimate the error in the approximation  $e^2 \approx P_3(1)$ .

4. (25 points) Consider the following procedure to estimate the value of  $\int_0^{0.1} \frac{\sin x}{x} dx$ .

(a) Determine the Maclaurin series for  $\frac{\sin x}{x}$ .

(b) Based on your series from part (a), determine the series for  $\int_0^{0.1} \frac{\sin x}{x} dx$ .

- (c) Estimate how accurately the first three non-zero terms of your series from part (b) approximates the value of the integral. You may leave your answer in terms of fractions, factorials, etc.