ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (35 points) Determine whether the following expressions converge or diverge. Give specific reasons for your answers.

   (a) $\displaystyle\int_3^\infty \frac{dx}{x\ln(x)}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(3n)!}{n!(n+1)!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

2. (35 points) Evaluate the following expressions.

   (a) $\displaystyle\int \frac{6x^2+2x+8}{x^3+x}\,dx$

   (b) $\displaystyle\int \frac{8}{(4x^2+1)^2}\,dx$

   (c) $\displaystyle\sum_{n=1}^{\infty} \Big(\tan^{-1}(n) - \tan^{-1}(n+1)\Big)$

   (d) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-1}{3+\sin x}\right)^n$

3. (30 points) Consider a thin plate of constant density δ (mass per unit area) in the first quadrant bounded by the line $x = a$ and the curve $y = \cosh x$.

   (a) Calculate the moment of the plate about the x-axis.

   (b) Calculate the moment of the plate about the y-axis.

4. (25 points) Consider the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(x-1)^n}{n\,2^n\ln(n)}$.

   (a) For what values of x does the power series converge absolutely?

   (b) For what values of x does the power series converge conditionally?

   (c) What is the interval of convergence?

5. (25 points) Estimate the error associated with using the first three non-zero terms of a series approximation for $\displaystyle\int_0^{0.1} \frac{\cos x - 1}{x^2}\,dx$. You do not need to simplify your answer to a decimal approximation. (Hint: start with a Maclaurin series of the integrand.)

6. (25 points) Find the surface area of revolution generated by revolving about the y-axis the curve $x(t) = t+\sqrt{2}$, $y(t) = (t^2/2) + \sqrt{2}\,t$, for $-\sqrt{2} \le t \le \sqrt{2}$.

7. (25 points) Consider the curve $r(\theta) = \cos\theta - 1$.

   (a) Carefully sketch the curve $r(\theta)$.

   (b) Find the area in the third quadrant inside the curve $r(\theta)$.

Formula Sheet

A short table of integrals. In the following, $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \quad$ for $a > 0$
2. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad$ for $u^2 < a^2$
3. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \quad$ for $u > a > 0$
4. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
5. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$
6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \quad$ for $u \neq 0$
7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \quad$ for $0 < u < a$
8. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \quad$ for $u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$
4. $\cosh^2 x - \sinh^2 x = 1$
5. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
6. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$

In formulas (3)–(6), x remains fixed as $n \to \infty$.

1. $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\displaystyle\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\displaystyle\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$
4. $\displaystyle\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
5. $\displaystyle\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x \quad$ (Any x)
6. $\displaystyle\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad$ (Any x)