

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (35 points) Determine if the following series converge absolutely, conditionally, or diverge. For any power series, determine the interval of convergence.

(a) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n} + \frac{1}{n^2} \right)$

(b) $\sum_{n=0}^{\infty} \left(\frac{1}{1+n^4} \right)$

(c) $\sum_{n=1}^{\infty} \frac{n+1}{2n + \frac{1}{n}}$

(d) $\sum_{n=1}^{\infty} a_n$ where $a_{n+1} = \frac{n!}{n^n} a_n$.

(e) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n} (x+1)^n$

2. (21 points) For each of the following, compute the series requested and determine its interval of convergence.

(a) The Taylor series around $x = 0$ for $\cosh x = \frac{e^x + e^{-x}}{2}$.

(b) The first three non-zero terms of the Taylor series around $x = 0$ for $(1+x)^{1/3}$.

(c) The first three non-zero terms of the Taylor series around $x = 0$ for $\sec^2 x$, given that $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$ for $|x| < \frac{\pi}{2}$.

3. (24 points) Consider the following steps for estimating the value of $\int_0^{0.1} \frac{dx}{1+x^4}$. **Please leave your answers in terms of factorials, fractions, etc.**

(a) Calculate first four non-zero terms of the Taylor series around $x = 0$ for $\frac{1}{1+x^4}$ and state its radius of convergence.

(b) Based on your series from part (a), determine the series approximation for the definite integral $\int_0^{0.1} \frac{dx}{1+x^4}$.

(c) If only the first three non-zero terms of the resulting series from part (b) are used, estimate the magnitude of the error.

4. (20 points) Consider the curve described by $4x^2 - 9y^2 - 16x - 20 = 0$.

(a) Determine the standard form of the curve, and classify the curve.

(b) Determine the coordinates of the foci.

(c) Determine the coordinates of the vertices.

(d) Determine the eccentricity.

(e) Sketch the curve and clearly label the foci and vertices.