ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A two-page crib sheet is allowed.

1. (21 points) Evaluate the following integrals.

   (a) $\int \frac{dx}{x+4}$ (b) $\int xe^{2x}\,dx$ (c) $\int \frac{x}{\sqrt{x^2+4x}}\,dx$

2. (21 points) Determine if the following integrals converge or diverge. Be sure to explain you answer.

   (a) $\int_0^9 \frac{dx}{(x-3)^2}$ (b) $\int_1^2 \frac{dx}{\ln x}$ (c) $\int_0^\infty \frac{\sin x}{1+x^2}\,dx$

3. (10 points) Decompose the rational function $\frac{3x^2+5x+3}{(x^2+1)(x+2)}$ using partial fractions.

4. (16 points) Consider the curve described by $y = e^{-x}$ for $0 \le x \le 1$. **Set up, but do not evaluate** the integrals to calculate the following:

   (a) The surface area generated by revolving the curve about the line $y = 1$.

   (b) The volume generated by revolving the curve about the line $x = 1$.

5. (21 points) Determine if the following sequences a_n converge or diverge. If the sequence converges, find its limit. Be sure to justify your answers.

   (a) $a_n = \cos^2(n\pi)$ (b) $a_n = (n+e)^{1/n}$ (c) $a_n = \ln(n^2+4) - \ln(n^2)$

6. (24 points) Determine if the following series converge absolutely, conditionally, or diverge. If the series converges, find its sum.

   (a) $\sum_{n=0}^{\infty} \left(\frac{e^n - e^{-n}}{e^n + e^{-n}}\right)$ (c) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

   (b) $\sum_{n=0}^{\infty} \frac{1}{\sqrt{(n+1)(n+2)}}$ (d) $\sum_{n=0}^{\infty} \frac{3^n + 4^n}{12^n}$

7. (16 points) For each of the following series determine the interval of convergence.

   (a) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ (b) $\sum_{n=0}^{\infty} n!\,(x-2)^n$

THERE IS MORE ON THE BACK

8. (21 points) Consider the following steps for estimating the value of $\int_0^{0.1} \sin(x^2)\,dx$. **Please leave your answers in terms of factorials, fractions, etc.**

   (a) Calculate first two non-zero terms of the Taylor series around $x = 0$ for $\sin(x^2)$.

   (b) Based on your series from part (a), determine an approximation for the definite integral $\int_0^{0.1} \sin(x^2)\,dx$.

   (c) If only the first term of the resulting series from part (b) is used, estimate the magnitude of the error.

9. (9 points) Find the first two non-zero terms of the Taylor series around $x = 0$ of the function $f(x) = \dfrac{\cos x - 1}{x^2}$.

10. (16 points) Consider the two curves described by $r_1(\theta) = 2(1 - \cos\theta)$ and $r_2(\theta) = 2\cos\theta$.

    (a) Sketch the curves $r_1(\theta)$ and $r_2(\theta)$.

    (b) **Set up, but do not evaluate**, the integral to calculate the arc length of $r_1(\theta)$.

    (c) Calculate $\dfrac{dy}{dx}$ on $r_1(\theta)$ when $\theta = \pi/2$.

    (d) **Set up, but do not evaluate**, the integral to compute the area inside $r_2(\theta)$ and outside $r_1(\theta)$.

11. (10 points) Consider the conic section $x^2 + \frac{\sqrt{3}}{2}xy + \frac{y^2}{2} + 5x - 6y - 5 = 0$.

    (a) Classify the conic section.

    (b) What angle of rotation is required to put this into the standard form for a conic section?

12. (15 points) Consider the conic section $x^2 + 9y^2 - 4x - 18y + 4 = 0$.

    (a) Determine the standard form of the curve, and classify the curve.

    (b) Determine the coordinates of the foci.

    (c) Determine the coordinates of the vertices.

    (d) Determine the eccentricity.

    (e) Sketch the curve and clearly label the foci and vertices.