

Exam 1 Review

1. Compute the following indefinite integrals:

$$\begin{array}{cccc}
 \int (1 - \cos x)^5 \sin x \, dx & \int \cos^2 x \sin^2 x \, dx & \int \frac{\theta^2 d\theta}{4 - \theta^2} & \\
 \int \tan \theta \, d\theta & \int \frac{ds}{5 + s^2} & \int \frac{dy}{y^2 - 2y + 2} & \int \frac{x^4}{x^2 + 4x + 4} \, dx \\
 \int \frac{\cos 3x}{\sqrt{5 + 2 \sin 3x}} \, dx & \int s^3 e^{-s^2} \, ds & \int \sqrt{y} \ln y \, dy & \int \frac{dz}{(4z^2 + 9)^2} \\
 & \int \frac{x + 10}{2x^2 + 5x - 3} \, dx & \int \frac{x^2}{\sqrt{25 - x^2}} \, dx &
 \end{array}$$

2. Use the definition of $\cosh x$ and $\sinh x$ to derive either

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

or

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y.$$

3. Find $\lim_{x \rightarrow 0} \frac{\sinh x}{x}$.
4. Show $y = \tanh 3x$ is a solution to $dy/dx = 3(1 - y^2)$.
5. Which of the following differential equations are linear, separable, both, or neither?

$$\begin{array}{cc}
 x^2 \frac{dy}{dx} + \cos x = y & \frac{dx}{dt} + xt = e^x \\
 x \frac{dx}{dt} + t^2 x = \sin t & 3t = e^t \frac{dy}{dt} + y \ln t \\
 (t^2 + 1) \frac{dy}{dt} = yt - y & 3r = \frac{dr}{d\theta} - \theta^3
 \end{array}$$

6. Solve

$$\begin{array}{l}
 \frac{dy}{dx} = 6x(y - 1)^{2/3}, \quad y(1) = 1, \\
 x^3 \frac{dy}{dx} + x^2 y = 2x^3 + 1, \quad y(1) = 1
 \end{array}$$

7. Find the area between the curves $y = \frac{x-9}{x^2-3x}$ and $y = \frac{x}{2}$ from $x = 1$ to $x = 2$.
8. Find the area between the curves $x = 4y^2$ and $x + 12y + 5 = 0$ from $y = -4$ to $y = 0$.
9. Find the volume remaining after a hole of radius 3 has been bored through a sphere of radius 5.
10. Set up the integrals for the volume of the ellipsoid generated by rotating the ellipse

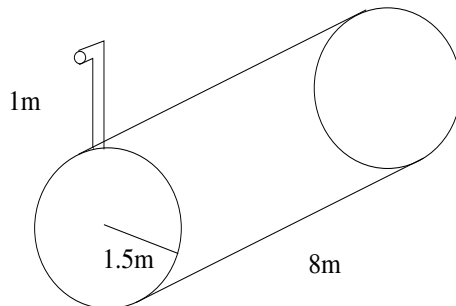
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

about the x-axis. Set up the integrals using both the disk/washer method as well as the shell method.

11. Find the surface area of the ellipsoid in problem 10.
12. Find the length of the parabola $y^2 = x$ from $(0,0)$ to $(1,1)$ and the length of the curve

$$y = \int_1^x \sqrt{3t^2 - 2} dt, \quad 1 \leq x \leq 4.$$

13. Find the surface area generated by revolving the curve $x = \sqrt{2y - y^2}$, $0 \leq y \leq 1$ about the y-axis.
14. The tank below is filled with water ($9800N/m^3$). Find the work required to pump



all the water out of the tank.

15. Suppose it took $2J$ of work to stretch a spring from its natural length of $30cm$ to $42cm$. How much work is needed to stretch it from $35cm$ to $40cm$?