

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (20 points) Determine whether the following series converge absolutely, conditionally, or diverge. Clearly justify your answer.

(a)  $\sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 3n}$

(c)  $\sum_{n=1}^{\infty} \frac{3^n n^3}{n!}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

2. (20 points) For each power series, determine the values of  $x$  for which the series converges absolutely, and converges conditionally. For each series, clearly state the radius of convergence.

(a)  $\sum_{n=0}^{\infty} \frac{2^n x^n}{(n+2)!}$

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n (2x-5)^n}{n}$

3. (20 points) Consider the following computations.

(a) Calculate the first four non-zero terms of the **Maclaurin** series for  $f(x) = (1+x)^{2/3}$ .

(b) Determine the (full) **Maclaurin** series for  $f(x) = x^2 \cos(x^2)$ .

4. (20 points) We wish to approximate  $\ln(1.1)$  by using the third-order Taylor polynomial  $P_3(x)$  of  $\ln(x)$  centered at  $a = 1$ .

(a) Calculate  $P_3(x)$ .

(b) Use  $P_3(x)$  to estimate  $\ln(1.1)$ . (You may leave your answer in terms of fractions.)

(c) Estimate the magnitude of the error associated with using  $P_3(x)$  to approximate  $\ln(1.1)$ .

5. (20 points) Using series, solve the initial value problem  $y' + 2y = 0$  where  $y(0) = 1$ .