ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show all of your work in your bluebook and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are not permitted. A one–page crib sheet is allowed.

1. (20 points) Do the following sequences $\{a_n\}_1^\infty$ converge or diverge? If a sequence converges and it is possible to determine its value, then do so. Be sure to explain your reasoning.

   (a) $a_n = \dfrac{n^3 + e^n}{2e^n + 1}$

   (b) $a_1 = \dfrac{1}{4}$ and $a_{n+1} = 2a_n$ for all $n \geq 1$

   (c) $a_{n+1} > a_n$ and $0 < a_n < 2$ for all $n \geq 1$

2. (20 points) Determine whether the following infinite series converge or diverge. Justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n^2 + 3n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$

3. (20 points) Determine where the following power series converge absolutely, converge conditionally, and diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} n^n (x-4)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (2x-1)^n}{n}$

4. (20 points) Consider the function $f(x) = e^{2x}$.

   (a) Starting from the definition, determine the Taylor series of $f(x)$ around $x = 1$.

   (b) Determine the interval of convergence for your series in part (a).

   (c) What are the first three non–zero terms of the Taylor polynomial around $x = 1$?

   (d) Estimate of the remainder of the polynomial in part (c) when $x = 1/2$. Leave your answer in terms of fractions, factorials, etc.

5. (20 points) Consider the following steps for evaluating $f(x) = \displaystyle\int_0^x \cos(t^2)\,dt$.

   (a) Determine the Taylor series for $\cos(t^2)$ around $t = 0$.

   (b) Using your series from part (a), calculate $f(x) = \displaystyle\int_0^x \cos(t^2)\,dt$.

   (c) What is the interval of convergence of your series in part (b)? Be sure to explain your reasoning.