ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show all of your work in your bluebook and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are not permitted. A two–page crib sheet is allowed.

1. (15 points) Set up, but do not evaluate, the integrals to find the center of mass of the region inside $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ when $x \geq 0$ and density $\delta = 1$.

2. (15 points) Solve the first order linear differential equation $\dfrac{dy}{dx}+\dfrac{y}{x}=x$ with the initial condition $y(1)=1$.

3. (20 points) Evaluate the following integrals.

   (a) $\displaystyle\int xe^{kx}dx$, k is constant (b) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$ (c) $\displaystyle\int_0^\infty \frac{dx}{(x-2)^2}$

4. (20 points) Consider the function $f(x)=\dfrac{1}{x^2(x^2+1)}$.

   (a) Find the partial fraction decomposition of $f(x)$ and evaluate all constants.

   (b) Now, compute $\displaystyle\int f(x)\,dx$.

5. (20 points) Determine whether the following integrals converge. Explain your reasoning.

   (a) $\displaystyle\int_1^\infty \frac{1}{x^3+1}\,dx$ (b) $\displaystyle\int_1^\infty \ln\left(1+\frac{1}{x}\right)dx$ (c) $\displaystyle\int_0^{\pi/2} \frac{1}{(\sin x)^{1/2}}\,dx$

6. (20 points) Investigate the *infinite sequences* $\{a_n\}_1^\infty$ where information about a_n is given below. Do the *infinite sequences* converge or diverge? Explain your reasons.

   (a) $a_n = \dfrac{n^2}{n+1}$

   (b) $a_n = \cos(n\pi)$

   (c) a_n has the following property: $|a_n| \leq \tan\left(\dfrac{1}{n}\right)$, for all $n \geq 1$.

7. (20 points) Explain whether the following *infinite series* converge or diverge. Make sure to state the name of the test you are using.

   (a) $\displaystyle\sum_{n=1}^\infty n^2e^{-n^3}$ (b) $\displaystyle\sum_{n=2}^\infty \frac{1}{\ln(n)}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{(n)!}{(2n)!}$

THERE IS MORE ON THE BACK

8. (10 points) Find the first three nonzero terms of the power series around $x = 0$ of the function $f(x) = \int_0^x \frac{1}{1+t^2}\,dt$.

9. (15 points) Consider the function $f(x) = \cos(2x)$.

   (a) Find the first two nonzero terms of the Taylor polynomial of $f(x)$ around $x = 0$.

   (b) Estimate the remainder of the polynomial in part (a) when $x = 1/2$.

10. (15 points) For the power series $\sum_{n=1}^{\infty} \frac{x^n}{3^n \sqrt{n}}$, find the interval of convergence and clearly indicate where the series converges absolutely, conditionally, and where it diverges.

11. (15 points) Consider the curve described by $r = 1 - \cos\theta$.

    (a) Set up and simplify, but do not evaluate, the integral to find the arc length of the curve for $0 \le \theta \le 2\pi$.

    (b) Set up and simplify, but do not evaluate, the integral to find the area enclosed by the curve.

12. (15 points) Consider the curve defined by $xy = 4$.

    (a) Characterize the curve.

    (b) Find the angle required to rotate this curve into standard form.

    (c) Write the equation defining the curve in standard form and sketch the curve. (Be sure to label appropriate points on the curve.)