APPM 1360 EXAM #1 Summer 2005

ON THE FRONT OF YOUR BLUEBOOK write your name and a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted.

1. (15 points) Evaluate the following statements as either TRUE or FALSE. No work need be shown for this problem. You must answer with the full word "TRUE" or "FALSE" – answers with only "T" or "F" will not be graded.

   a) The differential equation $\frac{dy}{dx} + xy = x$ is separable.

   b) The length of the curve $y = \left(\frac{x}{3}\right)^{\frac{3}{4}}$ from $x = 0$ to $x = 3$ is given by the integral:

$$L = \int_0^3 \sqrt{1 + \frac{1}{16}\left(\frac{x}{3}\right)^{\frac{-1}{2}}}\, dx$$

   c) The equation $\tanh^2 x = 1 + \operatorname{sech}^2 x$ is valid for all x.

2. (16 points) Find the area of the region enclosed by the curves $2x^2 - 2y = 0$ and $3x^2 - y = 8$.

3. (18 points) Evaluate the following expressions:

   a) $\frac{d}{dx}\left[\sin(\cosh(x))\right]$

   b) $\int \coth(2x)\, dx$

   c) $\int \frac{dx}{\sqrt{9x^2 - 4}}$

4. (24 points) Consider the region bounded on the left by $y = x^2$, on the right by $y = 2 - x$ and below by the x-axis. Set up the integrals (but do not integrate) to find the volumes of the following solids of revolution about the indicated line. Do one of the two parts using the disk/washer method and the other using the shell method.
   a) Revolving the region about the line $x = -2$.
   b) Revolving the region about the y-axis.

5. (15 points) Consider the function

$$y = \int_{\pi}^{x} \sqrt{t^2 + 8t + 15}\, dt$$

Find the length of the above curve y from $x = 4$ to $x = 7$.

6. (12 points) In electric circuits where both a resistor and an inductor are present (called an RL circuit), along with an applied voltage, Kirchhoff's Loop Rule states:

$$L\frac{dI}{dt} + RI = V$$

where L is the inductance, R is the resistance, V is the voltage and I is the current of the circuit. Derive an expression for the current (as a function of time t) in a RL series circuit in which the resistance, inductance, and voltage are constant. Assume the initial current is zero. *(Hint: Notice that the above equation is a first-order differential equation for I and remember that L, V and R are all constants).*