

ON THE FRONT OF YOUR BLUEBOOK write your name and a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted.

1. (20 points) Find the center of mass of a thin plate covering the region between the x-axis and the curve $y = \frac{2}{x^2}$, $1 \leq x \leq 2$, if the plate's density at the point (x,y) is $\delta(x) = x^2$.
2. (20 points)
 - a) Set up the partial fraction decomposition form for $\frac{9x^2 - 7x + 2}{(x^2 + 1)(x^2 - 1)^2}$, but do not solve for the constants A, B , etc.
 - b) We have used the fact that $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$ a number of times so far this semester (it even appears on your formula sheet). Show that the above formula holds by using an appropriate trigonometric substitution.
3. (20 points) For each sequence $\{a_n\}$ below, determine whether the sequence diverges or converges. If the sequence converges, state the limit of the sequence. Be sure to fully support your answer.

a) $a_n = \frac{\sin(n^6 + 4)}{n^2}$

b) $a_n = \frac{5^n}{n^2 - 7n}$

c) $a_n = n^{\frac{1}{n}}$

4. (16 points) Determine whether the following improper integrals converge or diverge.

a) $\int_1^{\infty} \frac{1}{9x^2 - 6} dx$

b) $\int_1^4 \frac{dx}{x \ln(x)}$

5. (24 points) Evaluate the following integrals:

a) $\int \ln(x) dx$

b) $\int \frac{6x}{\sqrt{x^2 + 9}} dx$

c) $\int \frac{2dx}{x^2 - 2x - 8}$

d) $\int xe^x dx$