

ON THE FRONT OF YOUR BLUEBOOK write your name and a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted.

1. (56 points) Determine whether the following series converge or diverge. Clearly and fully justify your answer, including stating which test you have used to reach your conclusion.

a) $\sum_{n=3}^{\infty} \frac{1}{n \ln(n)}$

b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

c) $\sum_{n=1}^{\infty} \frac{4n^{11} - 3}{5n^{12} - 7n^2 + 22}$

d) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n}$

e) $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$

f) $\sum_{n=6}^{\infty} \frac{\cos^4(2n)}{\sqrt[4]{n^5}}$

g) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

2. (12 points) A basketball is dropped from a height of 100 meters. Suppose that each time the ball drops h meters, it bounces back to a height of $0.5h$ meters.
- Find an expression that describes the total distance the ball has traveled. (*Hint: It may be helpful to draw out what is happening as the ball continues to bounce multiple times. Also, remember that the ball travels upwards and downwards as it bounces.*)
 - Does the sum describing the total distance the ball has traveled converge or diverge? Why? If it converges, find the sum.

3. (12 points) Suppose you want to approximate the value of $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{(2n)!}$ by only using the first four terms of the series. Give the value of this approximation, an error estimate for this approximation and explain your reasoning.

4. (20 points) For each of the following power series, determine the values of x for which the series converges absolutely and conditionally. Also, state the radius of convergence for each series.

a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n n!(x-4)^{n+1}}{3^n}$