

ON THE FRONT OF YOUR BLUEBOOKS WRITE: (1) your name, (2) your student ID number, (3) lecture section, (4) instructor's name and (5) a grading table. Write all of your work in your bluebook and box your final answer. A correct answer with no supporting work may receive zero credit, while an incorrect answer with supporting work may receive partial credit. Only one 8.5×11 formula-sheet is permitted. Use of calculators, class notes, any calculus book, etc. is not permitted.

1. (36 points, 6 each): Evaluate; simplify answers as appropriate:

$$\begin{array}{ll} \text{(a)} \frac{d}{dx} e^{\ln(2x)} & \text{(d)} \int e^{kx} dx; k = \text{constant} \\ \text{(b)} \frac{d}{dt} [t^2 \ln(1/t)] & \text{(e)} \int_e^{e^2} \frac{1}{x \ln x} dx \\ \text{(c)} \frac{d}{dx} (\sinh^2 x + \cosh^2 x) & \text{(f)} \int_{\frac{\pi}{2}}^{\pi} \sqrt{1 - \sin^2(\theta)} d\theta \end{array}$$

2. (24 points, 8 each): Consider the region enclosed between $y = x$ and $y = x^2$. Set up, but **DO NOT** evaluate, the resulting integrals.

- Find the enclosed area.
- Revolve the region around the y -axis. Use washers to find the volume.
- Use shells to find the volume in part (b) above.

3. (8 points): Find the length of the curve $y(x) = \int_{-2}^x \sqrt{9t^4 - 1} dt$, $-2 \leq x \leq -1$.

4. (12 points): Set up the integral as indicated below. **DO NOT** evaluate the resulting integral. Determine the center of mass of a thin plate of constant density in the form of a circle $x^2 + y^2 = 1$ in the first quadrant ($x \geq 0, y \geq 0$).

5. (12 points, 6 each): Consider the following differential equation: $\frac{dy}{dx} = \frac{y}{x} + k$, $k = \text{constant}$.

- Find the general solution when $k = 0$.
- When $k \neq 0$ find the solution to the equation that also satisfies the condition $y(1) = 0$.

6. (8 points): The function $y = \tanh^{-1}x$ for $|x| < 1$ can be written in terms of the natural logarithm function. *Derive* this relationship.