APPM 1360 EXAM 2 Fall 2005

ON THE FRONT OF YOUR BLUEBOOKS WRITE: (1) your name, (2) your student ID number, (3) lecture section, (4) instructor's name and (5) a grading table. Write all of your work in your bluebook and box your final answer. A correct answer with no supporting work may receive zero credit, while an incorrect answer with supporting work may receive partial credit. Only one 8.5 × 11 formula-sheet is permitted. Use of calculators, class notes, any calculus book, etc. is not permitted.

1. (*32 points*): Evaluate the following integrals; simplify answers as appropriate:

   (a) $\int te^{kt}$, $k = constant$ (d) $\int \frac{dx}{x^2 - x}$

   (b) $\int \frac{x^2}{x+1} dx$ (e) $\int \frac{dx}{x^2\sqrt{x^2-1}}$, $x > 1$

2. (*8 points*): Decompose the rational function $f(x) = \frac{x^2+1}{x(x^2+4)^2(x-1)^3}$ into partial fractions. DO NOT SOLVE FOR THE CONSTANTS.

3. (*15 points*): Determine if the following integrals converge. If the integral converges, calculate its value. Be sure to give supporting work for your answer.

   (a) $\int_0^2 \frac{dx}{(x-1)^{4/3}}$ (b) $\int_0^1 \ln(2x)\, dx$

4. (*21 points*): Find whether the sequences $\{a_n\}_1^\infty$ with a_n given below converge or diverge. Explain your answer.

   (a) $a_n = (-1)^n$ (b) $a_n = \left(1 - \frac{1}{n^2}\right)^n$ (c) $0 < a_n < 2,\ a_n < a_{n+1}$

5. (*24 points*):

   (a) Determine if the series below converge or diverge. If the series converges, find the sum. Be sure to give supporting work for your answer.

   (i) $\sum_{n=0}^{\infty} (-1)^n x^{2n}$, $x = constant$ (ii) $\sum_{n=1}^{\infty} \cos(\pi n)$

   (b) Determine if the series below converge or diverge. Be sure to give supporting work for your answer.

   (iii) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ (iv) $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

Answer Key for Exam A

(a) (*32 points*): Evaluate the following integrals; simplify answers as appropriate:

$$\text{(a)} \int te^{kt}, \quad k = constant \quad \text{(d)} \int \frac{dx}{x^2 - x}$$

$$\text{(b)} \int \frac{x^2}{x+1} dx \qquad \text{(e)} \int \frac{dx}{x^2\sqrt{x^2-1}}, \quad x > 1$$

(b) (*8 points*): Decompose the rational function $f(x) = \dfrac{x^2+1}{x(x^2+4)^2(x-1)^3}$ into partial fractions. DO NOT SOLVE FOR THE CONSTANTS.

(c) (*15 points*): Determine if the following integrals converge. If the integral converges, calculate its value. Be sure to give supporting work for your answer.

$$\text{(a)} \int_0^2 \frac{dx}{(x-1)^{4/3}} \qquad \text{(b)} \int_0^1 \ln(2x)\, dx$$

(d) (*21 points*): Find whether the sequences $\{a_n\}_1^\infty$ with a_n given below converge or diverge. Explain your answer.

$$(a)\ a_n = (-1)^n \quad \text{(b)}\ a_n = \left(1 - \frac{1}{n^2}\right)^n \quad \text{(c)}\ 0 < a_n < 2,\ a_n < a_{n+1}$$

(e) (*24 points*):

(a) Determine if the series below converge or diverge. If the series converges, find the sum. Be sure to give supporting work for your answer.

$$\text{(i)} \sum_{n=0}^{\infty} (-1)^n x^{2n}, \quad x = constant \qquad \text{(ii)} \sum_{n=1}^{\infty} \cos(\pi n)$$

(b) Determine if the series below converge or diverge. Be sure to give supporting work for your answer.

$$\text{(iii)} \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \qquad \text{(iv)} \sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$$