APPM 1360 EXAM 3 Fall 2005

ON THE FRONT OF YOUR BLUEBOOKS WRITE: (1) your name, (2) your student ID number, (3) lecture section, (4) instructor's name and (5) a grading table. Write all of your work in your bluebook and box your final answer. A correct answer with no supporting work may receive zero credit, while an incorrect answer with supporting work may receive partial credit. Only one 8.5×11 formula-sheet is permitted. Use of calculators, class notes, any calculus book, etc. is not permitted.

1. (15 points): Determine whether or not the *sequences* $\{a_n\}_{n=1}^{\infty}$ with a_n given below converge or diverge. Explain your reasoning.

$$\text{(a) } a_n = (-2)^n \ ; \quad \text{(b) } a_n = \frac{n}{e^n} \ ; \quad \text{(c) } a_n = \ln(n) - \ln(n+1)$$

2. (15 points): Determine whether or not the *series* given below converge or diverge. Explain your reasoning.

$$\text{(a) } \sum_{n=1}^{\infty} \frac{n}{n+1} \ ; \quad \text{(b) } \sum_{n=2}^{\infty} \frac{1}{n\,[\ln(n)]^2} \ ; \quad \text{(c) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$$

3. (20 points): Find the interval of convergence for the following two series:

$$\text{(a) } \sum_{n=1}^{\infty} \frac{x^n}{n}; \quad \text{(b) } \sum_{n=0}^{\infty} \frac{(2x)^n}{(2n)!}$$

4. (20 points): Find the Taylor series (power series) of the functions below about the point $x = 0$. Give supporting work and explanations. Determine the interval of convergence.

$$\text{(a) } f(x) = \frac{1}{2}\left(e^{2x} + e^{-2x}\right); \quad \text{(b) } f(x) = \frac{1}{1+x^2}$$

5. (30 points):

(a) Find the Taylor series (power series) of $f(x) = \int_0^x \cos(t^2)\,dt$ about $x = 0$.

(b) How many terms are needed to estimate the value of $\int_0^{1/2} \cos(t^2)\,dt$ with an error less than 10^{-2} in magnitude? Explain your reasoning.

(c) Evaluate the limits below *using Taylor series* (not L'Hopital's rule).

$$\text{(c1) } \lim_{x \to 0} \frac{e^{x^2} - e^{-x^2}}{x^2} \ ; \quad \text{(c2) } \lim_{x \to 0} \frac{\sin(x) + 1 - e^x}{x^2}$$