APPM 1360 FINAL EXAM Fall 2005

ON THE FRONT OF YOUR BLUEBOOKS WRITE: (1) your name, (2) your student ID number, (3) lecture section, (4) instructor's name and (5) a grading table. Write all of your work in your bluebook and box your final answer. A correct answer with no supporting work may receive zero credit, while an incorrect answer with supporting work may receive partial credit. *Only one* 8.5×11 *formula-sheet is permitted. Use of calculators, class notes, any calculus book, etc. is not permitted.*

1. (15 points): Evaluate

$$\text{(a) } \frac{d}{dx}\left[\sinh(x^2)\right]^2 \quad \text{(b) } \int_1^{e^3} \frac{[\ln(x)]^2}{x}dx \quad \text{(c) Solve } \frac{dy}{dx} = \frac{y}{x},\ y(1) = 2$$

2. (20 points): Set up, but *DO NOT EVALUATE*, the integrals to find:
   (a) The area enclosed in the first quadrant below the curve $y = x$ and inside the circle $x^2 + y^2 = 1$.
   (b) The center of mass of the circle $x^2+y^2 = r^2$, $r =$ constant, with density $\delta(x) = x$ *in the first quadrant.*

3. (30 points): Evaluate

$$\text{(a) } \int x\sin(x)\,dx \quad \text{(b) } \int \frac{x^2}{\sqrt{1-x^2}}\,dx \quad \text{(c) } \int \frac{dx}{x^2 - x}$$

4. (20 points): Determine if the following integrals converge. Explain your reasons.

$$\text{(a) } \int_1^{\infty} e^{-x^2}dx \quad \text{(b) } \int_0^2 \frac{dx}{(x-1)^2}\,dx$$

5. (30 points): Determine if the following *sequences* $\{a_n\}_1^{\infty}$ converge or diverge. Explain your reasons.

$$\text{(a) } a_n = \frac{\ln(n)}{n} \quad \text{(b) } a_n = \sec(n\pi) \quad \text{(c) } |a_n| \le 2, \text{for all } n$$

6. (15 points): Determine if the following *series* converge or diverge. Explain your reasons.

$$\text{(a) } \sum_{n=1}^{\infty} x^{2n},\ x = \text{constant} \quad \text{(b) } \sum_{n=1}^{\infty} \frac{n}{n^3+1} \quad \text{(c) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/2}}$$

THERE IS MORE ON THE OTHER SIDE

7. (20 points): Determine the Taylor series around $x = 0$ for the following functions and find the interval of convergence. Explain your reasons.

$$\text{(a) } f(x) = 1 + x^2 \quad \text{(b) } f(x) = x\sin(x) \quad \text{(c) } f(x) = \int_0^x \frac{dt}{1-t}$$

8. (20 points): Determine the Taylor series around $x = 0$ for $f(x) = \int_0^x \sin(t^2)dt$. Estimate the value of $f(0.1)$ with an absolute error no larger than 0.001.

9. (20 points): Given $2x^2 + 6xy + 2y^2 = 1$,

(a) Characterize the curve, i.e. what type of conic section is it. Explain.

(b) Find the angle which is needed to rotate this curve into standard form.

(c) Find the equation in the rotated coordinates.

(d) Sketch the curve in the original coordinates. Label the vertices, foci and other important features.

10. (10 points): Find the area outside $r = 1$ and inside $r = 1 - \cos(\theta)$.