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Final 1360 12/3/05 (1)

Ans

1. 15
2. 20
3. 30
4. 20
5. 30
6. 15
7. 20
8. 20
9. 20
10. 10

200 ✓✓

100

1) a) $\frac{d}{dx}(\sinh x^2)^2 = 2\sinh x^2 \cdot \cosh x^2 \cdot 2x = 4x \sinh x^2 \cosh x^2$

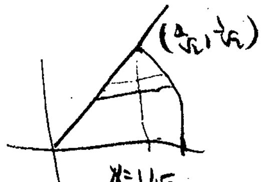
b) $\int_1^{e^3} \frac{(\ln x)^2}{x}\,dx = \int_1^{e^3} \frac{d}{dx}\frac{(\ln x)^3}{3}\,dx = \frac{(\ln x)^3}{3}\Big|_1^{e^3} = \frac{(\ln e^3)^3}{3}$

$= \frac{3^3}{3} = 9$

c) $\frac{dy}{dx} = \frac{y}{x}$. $\ln y = \ln x + C$ $y = Ax$ $y(1)=2$

$y = 2x$

2) a) 20



$A = \int_0^{1/\sqrt2} (\sqrt{1-y^2} - y)\,dy$

$y = x$, $x^2+y^2=1$ $2y^2=1$ $y = 1/\sqrt2$

or $A = \int_0^{1/\sqrt2} x + \int_{1/\sqrt2}^1 \sqrt{1-x^2}\,dx$

ALT

$A = \frac12 \int_{\pi/4} r^2\,d\theta$

$= \frac12 \int_0 1\,d\theta$

b)

$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm}$ $\bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$

$dm = \delta\,dA = x\sqrt{r^2-x^2}\,dx$ $dA = y\,dx$

$\bar{x} = \frac{\int_0^r x \cdot x\sqrt{r^2-x^2}\,dx}{\int_0^r x\sqrt{r^2-x^2}\,dx}$, $\bar{y} = \frac{\frac12\int_0^r y\,x\,y\,dx}{\int_0^r x\sqrt{r^2-x^2}\,dy}$

$= \frac{\frac12 \int_0^r x(r^2-x^2)\,dx}{\int_0^r x\sqrt{r^2-x^2}\,dx}$

3\. a. $\int x \sin x = -x\cos x - \int -\cos x\,dx$

$= -x\cos x + \sin x + C$

$\frac{d}{dx}(\quad) = +x\sin x - \cos x + \cos x = x\underline{\sin x}$ ✓

b) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2\theta}{\cancel{\cos\theta}}\,\cancel{\cos\theta}\,d\theta$

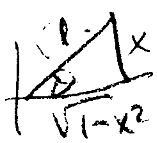
$x = \sin\theta$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$

$= \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$

$= \frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2} + C.$



c) $\int \frac{dx}{x(x-1)}$

$\frac{1}{x(x-1)} = \frac{A}{x-1} + \frac{B}{x}$

$1 = Ax + B(x-1)$

$B = -1 \quad A = 1$

$= \int\left(\frac{1}{x-1} - \frac{1}{x}\right)$

$= \underline{\ln x-1 - \ln x + C}$

4\. a) $\int_1^\infty e^{-x^2}\,dx \le \int_1^\infty e^{-x}\,dx$ conv.

b) $\int_0^2 \frac{dx}{(x-1)^2}$ $p = 2$ finite point div.

5. a) $\frac{\ln n}{n} \to 0$ conv.

b) $\sec n\pi = \frac{1}{\cos n\pi} = (-)^n$ DNE Div

c) $|a_n| \le 2$ not enough info
may or may not.

6. a) $\sum_1^\infty x^{2n}$ conv. $|x|<1$ Div. $|x| \ge 1$

b) $\sum_1^\infty \frac{n}{n^3+1}$ compare $\sum \frac{1}{n^2}$ $p=2$ conv.

c) $\sum_1^\infty \frac{(-)^n}{n^{1/2}}$ conv. AST.

7. a) $1+x^2$ conv. for all x

b) $x \sin x = x \sum_0^\infty (-)^n \frac{x^{2n+1}}{(2n+1)!} = \sum_0^\infty (-)^n \frac{x^{2n+2}}{(2n+1)!}$

$\cos x$

Use $\cos x$

$x \cos x = x \sum_0^\infty (-)^n \frac{x^{2n}}{(2n)!} = \sum_0^\infty (-)^n \frac{x^{2n+1}}{(2n)!}$ $|x| < \infty$

c) $\frac{1}{1-x} = \sum_{n=0}^\infty x^n$

$\int_0^x \frac{1}{1-t} = \sum_0^\infty \int_0^x t^n \, dt = \sum_0^\infty \frac{x^{n+1}}{n+1}$ $-1 \le x < 1$
AST Div

BTW $= -\ln(1-x)$

(4)

8. $\sin x = \sum_0^\infty (-)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} \cdots$

$\int_0^x \sin t^2 \, dt = \sum_0^\infty (-)^n \int_0^x \frac{t^{4n+2}}{(2n+1)!}$

$\int_0^x \sin t^2 \, dt = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} \cdots$

$= \sum_0^\infty (-)^n \frac{x^{4n+3}}{(4n+3)(2n+1)!}$ $\quad |x| < \infty$

$= \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!}$

Find interval of conv.

$\underline{|x| < \infty}$

$\int_0^x \sin t^2 \, dt = \frac{x^3}{3} + R_3$

$|R_3| \le \frac{|x|^7}{7 \cdot 3!} = \frac{10^{-7}}{7 \cdot 3!} < 10^{-5}$

10^{-5}

$\int_0^1 \sin t^2 \, dt = \frac{10^{-3}}{3}$

9. $2x^2+6xy+2y^2=1$

a) $A=C=2 \quad B=6$

$\Delta = B^2-4AC = 36-4\cdot 4 > 0$

Hyperbola

b) $\tan 2\alpha = B/(A-C) = \frac{6}{0}$ $\quad \boxed{\alpha = \pi/4}$

c) $x = \frac{x'-y'}{\sqrt{2}} \qquad y = \frac{x'+y'}{\sqrt{2}}$

$\frac{2}{2}(x'^2 - \cancel{2x'y'} + y'^2) + \frac{6}{2}(x'^2 - y'^2) + \frac{2}{2}(x'^2 + \cancel{2x'y'} + y'^2) = 1$

$5x'^2 - y'^2 = 1 \qquad a^2 = \frac{1}{5},\ b^2 = 1$

$c^2 = 1 + \frac{1}{5} = 6/5 \qquad c = \sqrt{6/5}$

Asympt. $y' = \pm\sqrt{5}x'$

$\Rightarrow x = \frac{(1\mp\sqrt{5})x'}{\sqrt{2}} \qquad y = \frac{(1\pm\sqrt{5})x'}{\sqrt{2}}$

$y = \left(\frac{1\pm\sqrt{5}}{1\mp\sqrt{5}}\right)x$

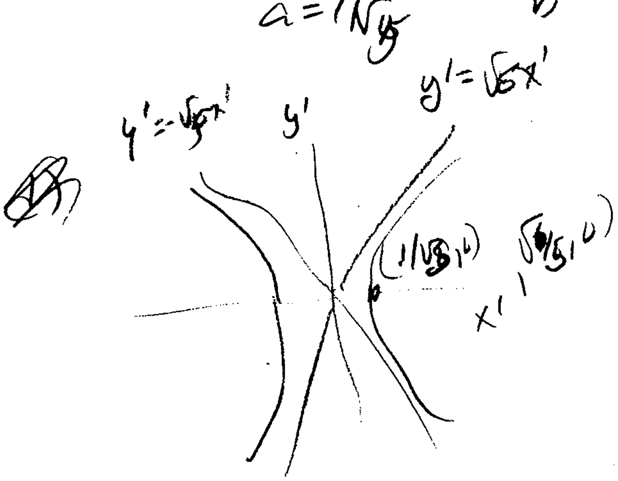
$y = \left(\frac{1+\sqrt{5}}{1-\sqrt{5}}\right)x = -\frac{3.2}{1.2}x$

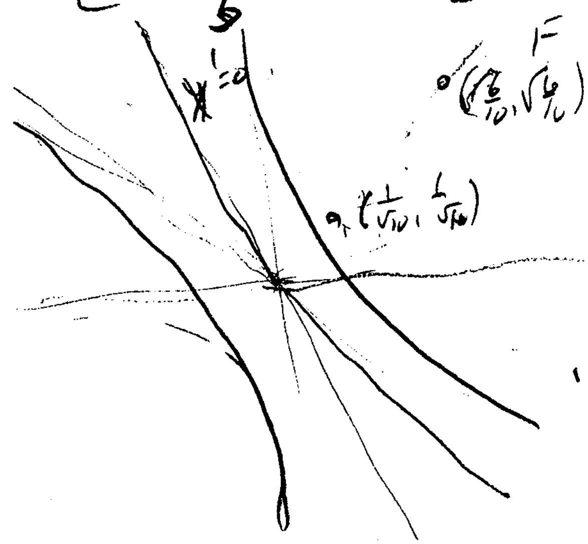
$y = \frac{1-\sqrt{5}}{1+\sqrt{5}}x = -\frac{1.2}{3.2}x$ asympt.

$a^2 = 1/5 \qquad b^2 = 1$

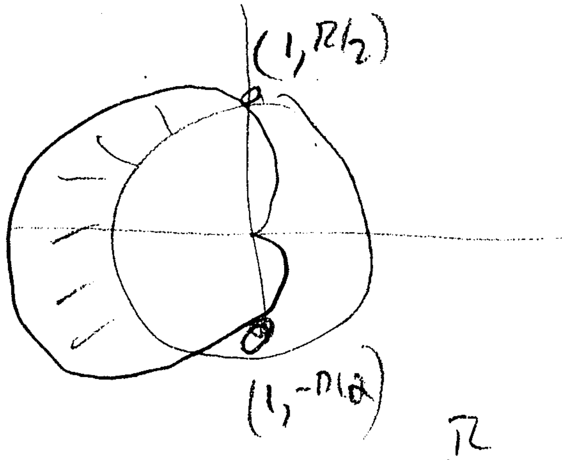
$a = 1/\sqrt{5} \qquad b = 1$

$c^2 = \frac{6}{5} \qquad C = \sqrt{\frac{6}{5}}$





10)



$r = 1$

$r = 1 - \cos\theta$

$\cos\theta = 0 \quad \theta = \pm \pi/2$

$$A = \frac{2}{2}\int_{\pi/2}^{\pi} \left((1-\cos\theta)^2 - 1^2 \right) d\theta$$

$$= \int_{\pi/2}^{\pi} (1 - 2\cos\theta + \underbrace{\cos^2\theta}_{\frac{1}{2}+\frac{1}{2}\cos 2\theta} - 1)$$

$$= \int_{\pi/2}^{\pi} (\tfrac{3}{2} - 2\cos\theta + \tfrac{1}{2}\cos 2\theta)$$

$$= \left(\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4} \right)\Bigg|_{\pi/2}^{\pi}$$

$$= \underbrace{\frac{3\pi}{2}}_{6\pi/4} - \left[\frac{3}{2}\left(\frac{\pi}{2}\right) - 2 \right]$$

$$= 2 + 3\pi/4$$