ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and crib sheets NOT permitted. No electronic devices may be used during the exam.

1. (20 points) Evaluate the following and simplify your answers.

   (a) $\dfrac{d}{dx}\sinh^{-1}(\tan x)$ (b) $\displaystyle\int_0^{\ln 2} e^{\cosh(2x)}\sinh(2x)\,dx$ (c) $\displaystyle\int \frac{dx}{\sqrt{36x^2+9}}$

2. (20 points)

   (a) Find the arc length of the curve defined by $y(x) = \displaystyle\int_1^x \sqrt{t^4-1}\,dt$ between $x = 1$ and $x = 4$.

   (b) Find the surface area generated by revolving about the x-axis the curve $y(x) = \cosh(x)$ for $x = 0$ to $x = \ln(2)$.

3. (20 points) Consider the region in the first quadrant bounded above by $y = x - x^2$.

   (a) Clearly graph (not sketch) the region.

   (b) Find the volume of the solid generated by revolving the curve about the x-axis.

   (c) Set up, *but do not evaluate*, the integral calculations necessary to determine the y-coordinate of the center of mass, $\bar{y}$. Clearly explain how you would then determine $\bar{y}$. You may assume the density ρ is constant.

4. (20 points) Consider the region bounded by $y = 2\sin x$ and $y = \sin(2x)$, for $0 \le x \le \pi$.

   (a) Clearly graph (not sketch) the functions. *If you cannot draw a graph of these functions, you may 'purchase' a sketch from us at a cost of 5 points.*

   (b) Set up, *but do not evaluate*, an integral to find the volume generated by revolving the region about the line y-axis.

   (c) Set up, *but do not evaluate*, an integral to find the volume generated by revolving the region about the line $x = -1$.

   (d) Set up, *but do not evaluate*, an integral to find the volume generated by revolving the region about the line $y = 3$.

5. (20 points) Solve the following differential equations.

   (a) $\dfrac{dy}{dx} = x^2 + x^2y^2$ where $y(0) = \dfrac{\pi}{4}$

   (b) $x\dfrac{dy}{dx} = \sinh(x) - y$

THERE IS SOME USEFUL INFORMATION ON THE BACK

A short table of integrals. In the following, $a \neq 0$.

1. $$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } a > 0$$

2. $$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u^2 < a^2$$

3. $$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u > a > 0$$

4. $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

5. $$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$$

6. $$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u \neq 0$$

7. $$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } 0 < u < a$$

8. $$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u^2 > a^2$$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$
4. $\cosh^2 x - \sinh^2 x = 1$
5. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
6. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$