APPM 1360 EXAM #2 Spring 2006

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and crib sheets NOT permitted. No electronic devices may be used during the exam.

1. (40 points) Evaluate the following integrals.

   (a) $\displaystyle\int \arcsin(x)\,dx$

   (b) $\displaystyle\int_2^3 \frac{dx}{x^2\sqrt{x^2-1}}$

   (c) $\displaystyle\int \frac{x^3+x^2}{x^2+x-2}\,dx$

   (d) $\displaystyle\int_0^3 \frac{1}{x-1}\,dx$

2. (14 points) Use direct integration, or comparison tests, to determine whether the following integrals converge or diverge. Be sure to fully explain your reasoning.

   (a) $\displaystyle\int_0^\infty \frac{1}{1+e^{-x}}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{x^3+x^{3/2}}{x^4+x^{1/2}}\,dx$

3. (18 points) Do the following sequences $\{a_n\}_1^\infty$ converge or diverge? For parts (a) and (b), if the sequence converges, find the value. Be sure to fully explain your reasoning.

   (a) $a_n = \left(\dfrac{n+3}{n}\right)^n$

   (b) $a_n = \dfrac{\ln(n^2)}{\ln(2n)}$

   (c) a_n satisfies $a_{n+1} \geq a_n$ and $0 < a_n < 4$

4. (14 points) Evaluate the following convergent infinite series.

   (a) $\displaystyle\sum_{n=0}^\infty \frac{5+2^n}{3^{n+1}}$

   (b) $\displaystyle\sum_{n=3}^\infty \frac{1}{n^2-n}$

5. (14 points) Do the following infinite series converge or diverge? Be sure to fully explain your reasoning.

   (a) $\displaystyle\sum_{n=1}^\infty (e^n+1)^{1/n}$

   (b) $\displaystyle\sum_{n=3}^\infty \frac{1}{n\,\ln(n)}$

THERE IS SOME USEFUL INFORMATION ON THE BACK

A short table of integrals. In the following, $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \quad$ for $a > 0$

2. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad$ for $u^2 < a^2$

3. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \quad$ for $u > a > 0$

4. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

5. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$

6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \quad$ for $u \neq 0$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \quad$ for $0 < u < a$

8. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \quad$ for $u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$
4. $\cosh^2 x - \sinh^2 x = 1$
5. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
6. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$