ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and crib sheets NOT permitted. No electronic devices may be used during the exam.

1. (15 points) Determine whether each statement is always TRUE, or FALSE, assuming each summation goes from $n = 1$ to $n = \infty$. You do not need to justify your answers, but you must write the complete word TRUE or FALSE to receive credit.
   (a) If $\sum |a_n|$ converges, then $\sum a_n$ converges.
   (b) If $\sum a_n$ converges, then $\sum |a_n|$ converges.
   (c) If $\sum a_n(6)^n$ converges, then $\sum a_n(-6)^n$ converges.
   (d) The Ratio Test can be used to determine whether $\sum(1/n^3)$ converges.
   (e) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

2. (15 points) Determine whether the following infinite series converge absolutely, converge conditionally, or diverge. Be sure to justify your answers.
   (a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n \ln(n)}{\sqrt[3]{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin(n)}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+e}{n}\right)^n$

3. (15 points) Consider the function $f(x) = e^x$.
   (a) Starting from the definition, derive the Taylor Series of $f(x)$ with a center of $a = 1$.
   (b) Now, using the first three non-zero terms of your series, estimate the value of e^2.

4. (20 points) Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n}}$.
   (a) What is the interval of convergence?
   (b) Where is the series absolutely convergent?
   (c) Where is the series conditionally convergent?

5. (15 points) Consider the function $f(x) = \ln(1+x)$. If you were to calculate its third derivative, you would find that $f'''(x) = \dfrac{2}{(1+x)^3}$. Now, suppose you were going to approximate the function by using the third order Taylor polynomial, that is $f(x) \approx P_3(x)$, with a center of $a = 0$. Estimate an upper bound for the magnitude of the error associated with the approximation $\ln(1/2) = f(-1/2) \approx P_3(-1/2)$.
   - Note that you are only asked to estimate the error, not the value of $\ln(1/2)$. Sweet!
   - Here's the bummer. You're not dealing with an alternating series for the given value of x.

6. (20 points) Consider the problem of calculating the value of the definite integral $\displaystyle\int_0^{0.1} \frac{1}{1+x^3}\,dx$.
   (a) First, determine the Maclaurin series of $\dfrac{1}{1+x^3}$.
   (b) Now, using your Maclaurin series from part (a), calculate the series for $\displaystyle\int_0^{0.1} \frac{1}{1+x^3}\,dx$.
   (c) Estimate an upper bound for the magnitude of the error if you used the first three non-zero terms of the series from part (b) to approximate the value of the definite integral.
   (d) Finally, is your approximation for the value of the definite integral too large, too small, or if it is not possible to determine, state "Not enough information." Explain your reasoning.

THERE IS SOME USEFUL INFORMATION ON THE BACK

A short table of integrals. In the following, $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } a > 0$

2. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u^2 < a^2$

3. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u > a > 0$

4. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

5. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$

6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u \neq 0$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } 0 < u < a$

8. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$
4. $\cosh^2 x - \sinh^2 x = 1$
5. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
6. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$

In formulas (3)–(6), x remains fixed as $n \to \infty$.

1. $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\displaystyle\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\displaystyle\lim_{n\to\infty} x^{1/n} = 1, \quad x > 0$
4. $\displaystyle\lim_{n\to\infty} x^n = 0, \quad |x| < 1$
5. $\displaystyle\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x, \quad |x| < \infty$
6. $\displaystyle\lim_{n\to\infty} \frac{x^n}{n!} = 0, \quad |x| < \infty$

Here are some common Maclaurin Series and the values of x for which they converge.

1. $\displaystyle\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1$

2. $\displaystyle\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-x)^n, \qquad |x| < 1$

3. $\displaystyle e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty$

4. $\displaystyle\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$

5. $\displaystyle\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \qquad |x| < \infty$

6. $\displaystyle\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \qquad -1 < x \leq 1$

7. $\displaystyle\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| \leq 1$

8. $\displaystyle(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots + \frac{m(m-1)(m-2)\cdots(m-n+1)}{n!}x^n + \cdots$
$\displaystyle= \sum_{n=0}^{\infty} \binom{m}{n} x^n, \qquad |x| < 1$