

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (18 points, 9 each) Evaluate the following integrals,

$$(a) \int \frac{x^4}{x^2 - 1} dx, \quad (b) \int \frac{x^2}{\sqrt{1 - x^2}} dx$$

2. (8 points) Set up, but do NOT evaluate the constants, the partial fraction decomposition for the function

$$f(x) = \frac{x^3 + 2}{(2x - 1)^3(x^2 + 9)(x^2 + x + 1)^2}$$

3. (14 points, 7 each) Determine whether the integrals converge or diverge. Explain your reasoning.

$$(a) \int_{-2}^2 \frac{1}{x - 1} dx, \quad (b) \int_0^{\infty} \frac{1}{\sqrt{x}(x + 1)} dx$$

4. (12 points, 6 each) Consider the following integral

$$I_n = \int_0^1 (\ln x)^n dx, \quad n \geq 0$$

(a) Find an equation relating I_n and I_{n-1} .

(b) It can be shown that

$$I_n = (-1)^n n!$$

Determine whether the sequence $\{I_n\}_{n=1}^{\infty}$ converges or diverges. Explain your reasoning.

5. (21 points, 7 each) Determine whether the sequences below converge or diverge. You must explain your reasoning.

(a) $\{\ln n - \ln(n + 1)\}_{n=1}^{\infty}$

(b) $\left\{ \frac{\sin n}{\sqrt{n}} \right\}_{n=1}^{\infty}$

(c) $\{a_n\}_{n=1}^{\infty}$ where for all $n \geq 1$, $a_{2n} = 0$, $a_{2n+1} > a_{2n-1}$, $a_1 = 1$ and $a_n < 2$.

6. (21 points, 7 each) Determine whether the series below converge or diverge. You must explain your reasoning.

$$(a) \sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n} + 1}, \quad (b) \sum_{n=2}^{\infty} \frac{1}{n \ln(n^2)}, \quad (c) \sum_{n=1}^{\infty} \sin\left(\frac{\pi}{4n}\right)$$

7. (6 points) Calculate the infinite sum: $0.4 + 0.16 + 0.064 + 0.0256 + \dots$

Good Luck!!!