

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (21 points, 7 each) Determine whether the sequences $\{a_n\}_{n=1}^{\infty}$ given below converge or diverge. You must explain your reasoning.

(a) $a_n = 2 \cos(n\pi)$,

(b) $a_n = \ln n + \ln(n + 1)$,

(c) a_n satisfies $0 < a_n < a_{n+1} < 1$ for all n .

2. (28 points, 7 each) Determine whether the series below converge or diverge. You must explain your reasoning.

(a) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$, (b) $\sum_{n=1}^{\infty} \ln(2n) - \sum_{n=1}^{\infty} \ln(4n + 2)$, (c) $\sum_{n=2}^{\infty} \sin\left(\frac{1}{2n}\right)$, (d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

3. (14 points, 7 each) Find the interval of convergence of the series given below.

(a) $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{n}$, (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}} (2x-3)^n$.

4. (a) (7 points) Find the first four nonzero terms of the power series of $f(x) = \sin x$ around $x = \frac{\pi}{4}$.

(b) (7 points) Find the power series of $f(x) = \frac{x^2}{1+x}$ around $x = 0$.

5. (a) (10 points) Find the power series associated with $\int_0^x \cos(2t) dt$.

(b) (10 points) Obtain an approximation to $\int_0^{1/2} \cos(2x) dx$ to less than 0.01 accuracy.

(c) (4 points) Find the exact value of the integral and verify that your approximation is valid up to the suggested accuracy. You may use $\sin(1) = 0.841471$.

6. (14 points, 7 each) Evaluate the limits using power series methods.

(a) $\lim_{x \rightarrow 0} \frac{\sin(2x) + 1 - e^{2x}}{x^2}$, (b) $\lim_{x \rightarrow 0} \frac{1 - \cosh x}{1 - \cos x}$.

Good Luck!!!