Final Exam APPM 1360 Fall 2006

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

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1. (*14 points, 7 each*) Evaluate and simplify your answers as appropriate:

$$\text{(a)}\ \frac{d}{dx}\left[(2x)^{2x}\right], \quad \text{(b)}\ \int \tanh(kx)\, dx,\ k \text{ is constant}$$

2. Consider the following differential equation and initial condition

$$\frac{df}{dx} + f = 2, \quad f(0) = 0$$

(a) (*10 points*) Solve the equation and apply the initial condition to find $f(x)$.

(b) (*5 points*) Find the limit of the solution as $x \to \infty$.

3. (*24 points, 8 each*) The region bounded by the curves $y = \pm\dfrac{4}{\sqrt{x}}$ and the lines $x = 1$ and $x = 4$ is revolved around the y-axis to generate a solid. Set up, but **DO NOT** evaluate, the resulting integrals.

(a) Sketch the plate and find the area enclosed by the two curves.

(b) Find the volume of the solid.

(c) Find the center of mass of a thin plate covering the region if the plate's density is $\delta(x) = cx$ where c is a constant.

4. (a) (*20 points, 10 each*) Evaluate the following integrals:

$$\text{(i)}\ \int_1^e x\ \ln x\, dx, \quad \text{(ii)}\ \int \frac{1}{x^2(x^2+1)}\, dx$$

(b) (*10 points*) Determine if the integral converges or diverges. If it converges determine its value.

$$\int_0^\infty \frac{1}{(x+a)^4}\, dx, \quad a \text{ is constant}$$

5. (*21 points, 7 each*) Determine whether the sequences $\{a_n\}_{n=1}^{\infty}$ given below converge or diverge. You must explain your reasoning.

(a) $a_n = n\sin(\pi n)$,

(b) $a_n = \ln(2n+1) - \ln(n+1)$,

(c) a_n satisfies $|a_n| \leqslant \sin\left(\dfrac{1}{n}\right)$ for all $n \geqslant 1$.

6. (*24 points, 8 each*) Determine whether the series below converge or diverge. You must explain your reasoning.

$$\text{(a)}\ \sum_{n=1}^{\infty} \frac{n^2}{n^3+1}, \quad \text{(b)}\ \sum_{n=2}^{\infty} \cos\left(\frac{1}{2n}\right), \quad \text{(c)}\ \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

More on the back; turn over the page.

7. (*10 points*) Find the interval of convergence of the series given below.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$$

8. (a) (*12 points*) Establish the minimum number of terms needed in the Taylor series around $x = 0$ of e^{-x} to evaluate $e^{-0.2}$ to 10^{-2} accuracy. Provide your approximation of $e^{-0.2}$.

(b) (*12 points*) Find the power series of $f(x) = \dfrac{1}{\sqrt{1+x^2}}$. Use this expression to evaluate $f^{(4)}(0)$.

9. (*24 points, 8 each*) Consider the curve described by the equation

$$x^2 + 2\sqrt{3}\alpha\, xy - y^2 = 4$$

(a) Carefully explain what type of conic section this curve is.

(b) Take $\alpha = 1$. Find the rotation angle needed in order to put the equation in standard form. Explain what you would need to do to put the equation in standard form, but **DO NOT** carry out the calculation.

(c) Take $\alpha = 0$. Sketch the curve in the xy–plane. Find and label the vertices, foci, directrices and other important features.

10. (*24 points, 6 each*) Let's write the number eight(8)! We will use the lemniscate $r = 1 - \cos(2\theta)$ and the two circles $r = \pm 2\sin\theta$; r and θ are the usual polar coordinates. See figures below.

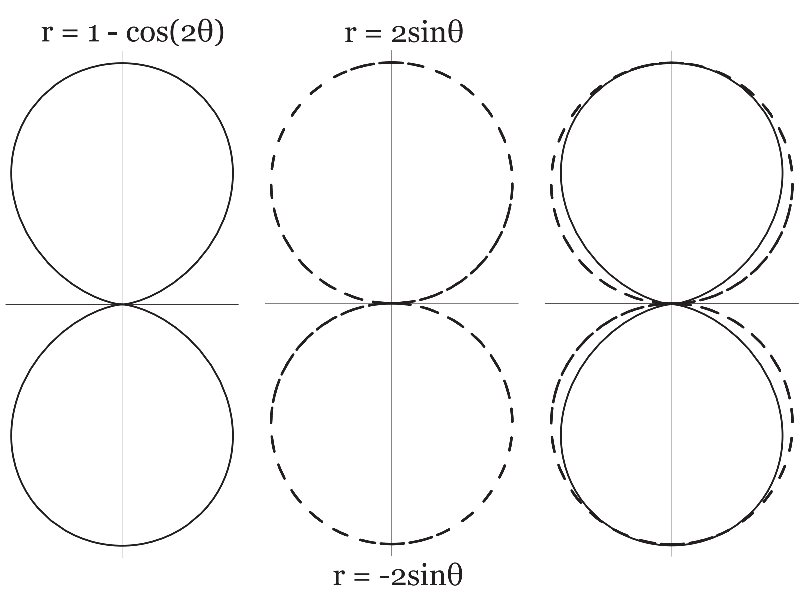
(a) Write the equation of the circle, $r = 2\sin\theta$, in cartesian xy–coordinates.

(b) Find the points where the two curves intersect.

(c) Set up, but **DO NOT** evaluate, the integrals describing the area inside the circles and outside the lemniscate.

(d) Set up, but **DO NOT** evaluate, the integrals describing the length of the two figures (the two circles and the lemniscate).

Extra credit: (*5 points: This problem is difficult; do it only if you have time available*). Establish, **WITHOUT** relying on the figures below, which figure requires less ink when writing eight.



Good Luck!!!