Exam #2 **APPM1360** **Spring 2007**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. Maximum score: 115 points.

Question 1: 16 points, **Question 2**: 18 points, **Question 3**: 24 points,
Question 4: 24 points, **Question 5**: 10 points, **Question 6**: 18 points

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1. Evaluate the following:

$$\text{(a)} \int \frac{1}{x^2\sqrt{x^2-1}}\,dx, \quad \text{(b)} \sum_{n=1}^{\infty}\left\{\frac{\cos n}{n^2} - \frac{\cos(n+1)}{(n+1)^2}\right\}$$

2. (a) Determine if the integral $\displaystyle\int_{-\infty}^{\infty} \frac{1+x}{1+x^2}\,dx$ converges or diverges.

(b) Calculate $\displaystyle\lim_{t\to\infty}\int_{-t}^{t} \frac{1+x}{1+x^2}\,dx$.

(c) Is the result in part (b) consistent with your answer in part (a)? Explain.

3. Determine whether the sequences $\{a_n\}_{n=1}^{\infty}$ below converge or diverge. You must explain your reasoning.

$$\text{(a) } a_n = \sqrt{\frac{2n}{n+1}}, \quad \text{(b) } a_n = \sinh(\ln n), \quad \text{(c) } a_n = \frac{n!}{2^n}$$

4. Determine whether the series below converge or diverge. You must explain your reasoning.

$$\text{(a)} \sum_{n=1}^{\infty} \frac{n^2}{(2n+1)(2n+5)}, \quad \text{(b)} \sum_{n=2}^{\infty} \frac{1}{n\ln(n^2)}, \quad \text{(c)} \sum_{n=1}^{\infty} \frac{(3n)!}{n!(n+1)!(n+2)!}$$

5. Write the first three terms of the series $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n}}$. Explain why the series converges and calculate the infinite sum.

6. (a) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{10}}$ converges absolutely, conditionally or diverges. You must explain your reasoning.

(b) How many terms in the series do we need in order to find the sum with error less than $4 \cdot 10^{-5}$? You may use $2^{10} = 1024$ and $3^{10} = 59049$.

Extra credit: (*5 points: This problem is difficult; do it only if you have time available*). Let $\{F_n\}$ be the Fibonacci sequence defined as $F_1 = 1$, $F_2 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geqslant 2$. Given the limit $\displaystyle\tau = \lim_{n\to\infty}\frac{F_{n+1}}{F_n}$ exists show that $\tau = (1+\sqrt{5})/2$.

Good Luck!!!