

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. Maximum score: 105 points.

Question 1: 18 points, **Question 2:** 16 points, **Question 3:** 20 points,
Question 4: 30 points, **Question 5:** 16 points

1. (a) Using *only* series and their properties evaluate the limit

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

- (b) Find the function with Mclaurin series

$$\sum_{n=1}^{\infty} nx^{n-1}$$

2. Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$$

3. (a) Write $f(x) = \frac{x}{(1-x)^2}$ as a power series.

- (b) Substitute $x = \frac{1}{2}$ in your answer in part (a) and thus, or otherwise, find $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

4. Consider the function $f(x) = \cos(\sqrt{x})$.

- (a) Find the Mclaurin series of $f(x)$.

- (b) If $f(x)$ is approximated by $1 - \frac{x}{2}$ estimate the error in the integral $\int_0^1 \cos(\sqrt{x}) dx$.

- (c) Calculate the exact value of $\int_0^1 \cos(\sqrt{x}) dx$. You may use the substitution $x = u^2$.

5. Classify the equation for the different values of c given below. You must explain your reasoning.

$$\frac{x^2}{15-c} - \frac{y^2}{c-6} = 1$$

- (a) $6 < c < 15$, (b) $c < 6$, (c) $c > 15$.

Extra credit (5 points): If $f(x) = \sin(x^3)$ find $f^{(15)}(0)$.

Good Luck!!!