Final Exam **APPM 1360** **Spring 2007**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. Maximum score: 215 points.

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1. (*14 points*) Evaluate and simplify as appropriate:

$$\text{(a) } \frac{d}{dx}[x\cosh(\ln x)], \quad \text{(b) } \int \frac{\sinh x}{1+\cosh x}\,dx$$

2. (*18 points*) The fish population $P(t)$ of a lake is attacked by a disease at $t = 0$ and it is changing according to the equation

$$\frac{dP}{dt} = -3\sqrt{P}$$

(a) If at time $t = 0$ there are 900 fish in the lake, find their population at any given time $t > 0$.

(b) If time is measured in weeks how long will it take for all the fish to vanish?

3. (*24 points*) Let R by the region between $y = x^2$ and $x = y^2$. Set up, but **DO NOT** evaluate, the resulting integrals.

(a) Sketch R and find its area.

(b) Find the volume of the solid formed by rotating R about the line $x = -1$.

(c) Find the center of mass of a thin plate covering R if the plate's density is $\delta(x) = \sqrt{x}$.

4. (a) (*20 points*) Evaluate the integral:

$$\int \frac{1}{x^2-1}\,dx$$

(i) using partial fractions, (ii) using an appropriate trigonometric substitution.

(b) (*10 points*) Consider the integral:

$$\int_0^\infty x\,e^{ax}\,dx$$

where a can take the values $a = 1$ and $a = -1$. Will the integral converge for either value of a, both or neither? You must explain your reasoning.

Extra credit: (*5 points: Attempt to do this only if you have time available*!) Find all values of a for which the integral converges.

5. (*21 points*) Determine whether the sequences $\{a_n\}_{n=1}^{\infty}$ given below converge or diverge. You must explain your reasoning.

$$\text{(a) } a_n = \frac{1+(-1)^n}{n}, \quad \text{(b) } a_n = 2^{\cos(n\pi)}, \quad \text{(c) } a_n = \frac{\ln n}{\sqrt{n}}$$

More on the back; turn over the page.

6. (*21 points*) Determine whether the series below converge or diverge. You must explain your reasoning.

$$\text{(a)} \sum_{n=1}^{\infty} \frac{n}{10n+17}, \quad \text{(b)} \sum_{n=1}^{\infty} \frac{\pi^n}{4^n}, \quad \text{(c)} \sum_{n=1}^{\infty} \frac{(n+2)!}{3^n (n!)^2}$$

7. (*10 points*) Find the interval of convergence of the series given below. Be sure to check the end–points.

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{\sqrt{2n+1}} x^n$$

8. (*24 points*) Using the Taylor series around $x = 0$ of $\cos(x)$:

(a) Determine the minimum number of terms needed to evaluate $\cos(1)$ to 10^{-2} accuracy.

(b) Provide an approximation for $\cos(1)$ with 10^{-2} accuracy.

(c) Find the series for $\sin(x)$ using the fact that $\sin(x) = -\frac{d}{dx}\cos(x)$.

9. (*24 points*) Consider the curve described by the equation

$$xy = 1$$

(a) Determine what type of conic section this curve is.

(b) Find the rotation angle needed in order to put the equation in standard form and show that its standard form is $x'^2/2 - y'^2/2 = 1$.

(c) Sketch $x^2/2 - y^2/2 = 1$ in the xy–plane. Find and label the vertices and the asymptotes.

10. (*24 points*) The spiral (see figure)

$$r = \theta$$

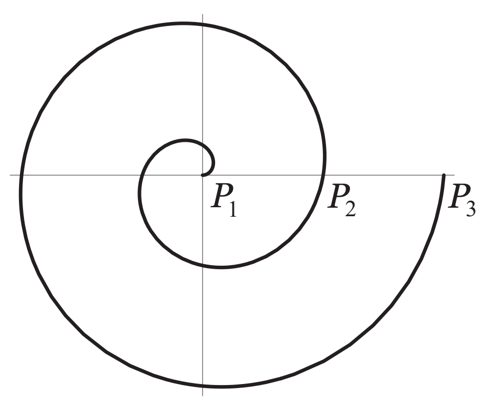
was studied by Conon, and later by Archimedes and it is called *Archimedes' spiral*. If Archimedes was able to study this curve in 225 BC then surely you can:

(a) Write its equation in cartesian coordinates. You may leave the equation in implicit form.

(b) Find the cartesian coordinates of the points P_1, P_2 and P_3, where the spiral passes through the positive x–axis.

(c) Find the area enclosed by the spiral between P_1 and P_2.

(d) Set up, but **DO NOT** evaluate, the integral describing the total ($P_1 \leqslant x \leqslant P_3$) length of the spiral.



Good Luck!!!