APPM 1360 Exam #3 Summer 2007

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) APPM 1360, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators, and electronics of any kind are NOT permitted.

1. (32 points) True or False: if the statement is *always* true, put **True** (not just T); if the statement is not *always* true, put **False** (not just F).

   (a) Suppose that $a_n > 0$ and $b_n > 0$, and that $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$. If you know that $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ must converge.

   (b) $\sum_{n=2}^{\infty} 5\left(\frac{1}{2}\right)^n$ converges to 2.5.

   (c) $\sum_{n=3}^{\infty} n e^{-n^2}$ converges.

   (d) $\sum_{n=5}^{\infty} \ln\left[\frac{n}{n+1}\right] = \ln(5)$

2. (33 points) Determine if the following converge or diverge. Explain your reasoning.

   (a) $\sum_{n=7}^{\infty} \frac{1}{\sqrt{n^2-4}}$ (b) $\sum_{n=11}^{\infty} \frac{4^{n+1}}{n^{10} 3^n}$ (c) $\sum_{n=13}^{\infty} \frac{\sin(n^2)}{n^{1.1}}$

3. (30 points) For the series $\sum_{n=17}^{\infty} \frac{(x-1)^{n+1}}{n 2^n}$, find: (a) The Radius of Convergence, (b) The Interval of Convergence, (c) the locations, if any, where it converges absolutely, and (d) the locations, if any, where it converges conditionally. State your reasoning clearly.

4. (15 points) Consider the (geometric) power series $\frac{1}{1+x^3} = \sum_{n=0}^{\infty} (-1)^n x^{3n}$.

   (a) Find a power series for $\frac{-3x^2}{(1+x^3)^2}$. (Hint: it's related to the power series for $\frac{1}{1+x^3}$)

   (b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{3n(-1)^n}{2^{3n-1}}$.

5. (15 points) Consider a shape made by the following process: Start with a sphere of radius 1. Place another sphere next to the first sphere, this one with radius $\frac{1}{\sqrt{2}}$. Place another sphere next to the second with radius $\frac{1}{\sqrt{3}}$, and so on so that the n^{th} sphere has radius $r_n = \frac{1}{\sqrt{n}}$.

   (a) Does the shape have finite volume? For a single sphere, $V = \frac{4}{3}\pi r^3$.

   (b) Does the shape have finite surface area? For a single sphere, $SA = 4\pi r^2$.