APPM 1360 Final Exam Summer 2007

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) "Final Exam" (3) APPM 1360, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators, and electronics of any kind are NOT permitted. There are 100 total points for the test (this will be scaled to 225 for the final grade).

1. (25 points) Set up, but **DO NOT EVALUATE** the integrals necessary to find:
   (a) (5pts) The volume generated by revolving the region bounded by $y=x\cos(x)$, $y=0$, $0\le x\le\frac{\pi}{2}$ about the line $x=\frac{\pi}{2}$.
   (b) (5pts) The function $x(t)$ which solves the differential equation $\frac{dx}{dt}+2\frac{x}{t}=\frac{1}{1-t^2}$.
   (c) (5pts) The length of the line parametrized by $x(t)=\cos(t)$, $y(t)=t+\sin(t)$, $0\le t\le\pi$.
   (d) (10pts) Find the solution (i.e. evaluate the integral(s)) to **ONE** of parts (a), (b), or (c).

2. (40 points) (a) (16pts) Show that the Taylor Series for $f(x)=\arctan(x)$ centered at $x_0=0$ is $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$.
   (b) (10pts) Find the interval of convergence for the series in part (a). Remember to check the endpoints.
   (c) (7pts) Estimate the value of $f(.1)=\arctan(.1)$ using the third order Taylor polynomial.
   (d) (7pts) Give an upper bound for the error made in the approximation in part (c). Clearly explain your reasoning.

3. (20 points, 10 each) (a) Put the equation $r=1+\cos(2\theta)$ in rectangular (Cartesian) coordinates. You do not need to simplify your answer.
   (b) Find the area enclosed by the curve $r=1+\cos(2\theta)$.

4. (10 points, 5 each) Do the following converge or diverge? Explain your reasoning.
   (a) $\sum_{n=3}^{\infty}\frac{n+1}{\sqrt{n^3-2}}$ (b) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

5. (5 points) Consider the numbers between 0 and 1. There are many. If I remove the middle third of these numbers, then only the numbers 0 through 1/3 and 2/3 through 1 remain:

0 ____________ 1/3 2/3 ____________ 1

Now imagine removing the middle third of each of the remaining intervals so that only 0 through 1/9, 2/9 through 1/3, 2/3 through 7/9, and 8/9 through 1 remain:

0 ____ 1/9 2/9 ____ 1/3 2/3 ____ 7/9 8/9 ____ 1

If I carry this process out ad infinitum, the total amount or length of numbers removed can be represented by a series: $\sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{2}{3}\right)^n$.

   (a) (5 points) Does this series converge? If so, then to what value?
   (b) (0 points: not a question for you to answer) It is a slightly paradoxical fact that there is an infinite quantity of numbers left over – even more than all of the integers put together.