Worksheet 1 Solutions

Find ~~the~~ $\cancel{\frac{dy}{dt}}$ the following derivatives

1) $\frac{dy}{dt}$: $y = \left(\frac{(t+1)(t-1)}{(t^2+2t)(t+3)} \right)^5$

$$\ln(y) = \ln \left(\frac{(t+1)(t-1)}{(t^2+2t)(t+3)} \right)^5$$

$$= 5\left[\ln(t+1) + \ln(t-1) - \ln(t^2+2t) - \ln(t+3) \right]$$

$$\frac{1}{y} \frac{dy}{dt} = 5\left[\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t^2+2t}[2t+2] - \frac{1}{t+3} \right]$$

$$\frac{dy}{dt} = 5\left[\frac{1}{t+1} + \frac{1}{t-1} - \frac{2t+2}{t^2+2t} - \frac{1}{t+3} \right] \left(\frac{(t+1)(t-1)}{(t^2+2t)(t+3)} \right)^5$$

2) $\boxed{\int \frac{1}{x[\ln(x)]^3} \, dx}$

Let $u = \ln(x)$

$du = \frac{1}{x} dx$

$dx = x \, du$

$$\int \frac{1}{u^3} \frac{x}{x} \, du$$

$$= \int u^{-3} \, du = \left. \frac{u^{-2}}{-2} \right|_{u=\ln(x)}$$

$$= -\frac{1}{2} \frac{1}{\ln(x)^2}$$

$\frac{+}{x_0} 280$

Hyperbolic functions

(3) $\int_0^{\ln 2} 4\, e^x \sinh x\, dx$ $\qquad \sinh x = \frac{e^x - e^{-x}}{2}$

$$\int_0^{\ln 2} 4 e^x \left[\frac{e^x - e^{-x}}{2}\right] dx = \int_0^{\ln 2} 4\left[\frac{e^{2x}}{2} - \frac{1}{2}\right] dx$$

$$= \int_0^{\ln 2} 2e^{2x} - 2\, dx = e^{2x}\Big|_0^{\ln 2} - 2x\Big|_0^{\ln 2}$$

$$= e^{2\ln(2)} - 1 - 2\ln(2) + 0$$

$$= e^{\ln(4)} - 1 - \ln(4) +$$

$$= 4 - 1 - \ln(4)$$

$$= 3 + \ln\left(\tfrac{1}{4}\right)$$

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(4) $\int \frac{1}{\sqrt{4+x^2}}\, dx = \sinh^{-1}\left(\frac{x}{2}\right) + C$

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5) $y = \cosh(x)$ $\qquad \frac{dy}{dx} = \sinh(x)$

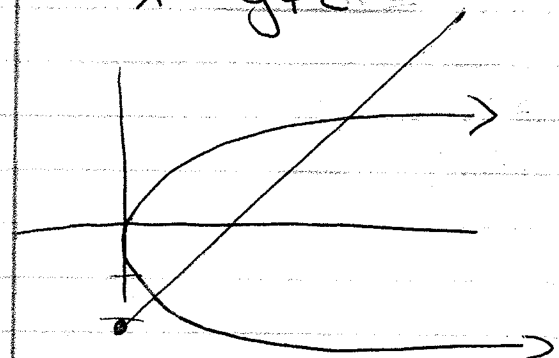
6) $y = \tanh(x^2)$ $\qquad \frac{dy}{dx} = \operatorname{sech}^2(x^2)\, 2x$

7) $y = \tanh^{-1}(\sin x)$ $\qquad \frac{dy}{dx} = \frac{1}{1-\sin^2 x} \cdot \cos x = \frac{\cos}{\cos}$ $\sec x$

Find the area between the curve

$x = y^2$

$x = y + 2$ $\qquad$ $x - 2 = y$



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$$y = \frac{1 \pm \sqrt{1+8}}{2}$$

$$y = \frac{1 \pm 3}{2}$$

$$y = \frac{4}{2}, \quad y = \frac{-2}{2}$$

$y = 2, -1$

$$\int_{-1}^{2} \cancel{x}\, y + 2 - y^2 \, dy$$

$$= \left. \frac{y^2}{2} + 2y - \frac{y^3}{3} \right|_{-1}^{2}$$

$$= \frac{4}{2} - \frac{1}{2} + 4 - 2 - \left[\frac{8}{3} + \frac{1}{3}\right]$$

$$= \frac{3}{2} + 2 - 3 = \frac{3}{2} - \frac{2}{2} = \boxed{\frac{1}{2}}$$

check

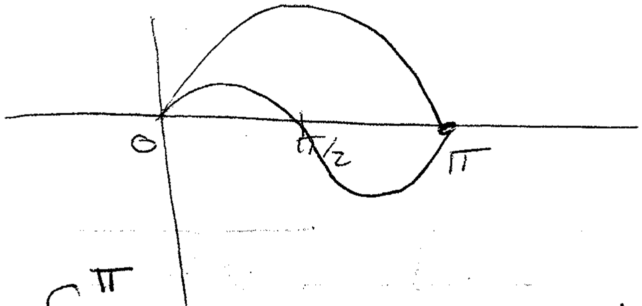
find the area between the two curves

$y = 2\sin x$

$y = \sin 2x \qquad 0 \leq x \leq \pi$

$2\sin x = 0$ at $0, \pi, 2\pi$

$\sin(2x) = 0$ at $0, \pi, 2\pi$



$$\int_0^{\pi} 2\sin x - \sin 2x \, dx$$

$$-2\cos x \Big|_0^{\pi} + \frac{1}{2}\cos(2x)\Big|_0^{\pi}$$

$$= -2[-1 \cancel{+} -1] + \tfrac{1}{2}[+1 - 1]$$

$$= 4$$

find the area between

$x = y^3 \qquad 0 < x < 1$

$x = y^2 \qquad 0 < y < 1$

$$\int_0^1 y^2 - y^3 \, dy = \frac{y^3}{3} - \frac{y^4}{4}\Big|_0^1$$

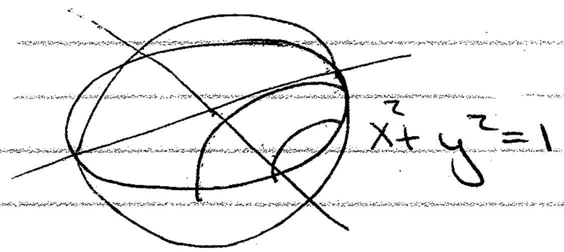
$$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

find the volume of the object when this is rotated around the y axis

$$\int_0^1 \pi\left[(y^2)^2 - (y^3)^2\right]dy = \int_0^1 \pi\left[y^4 - y^6\right]dy = \pi \frac{y^5}{5} - \frac{y^7}{7}\Big|_0^1$$

$$= \pi\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{7-5}{35} = \boxed{2/35}$$

$$V = \int_a^b A(x)\,dx$$



$$\text{Area}(x) = \pi \cdot \text{radius}^2$$

$$= \pi(1-x^2)$$

$$\int_{-1}^{1} \pi(1-x^2)\,dx$$

$$= \pi x - \frac{x^2}{2}\Big|_{-1}^{1}$$

$$= \pi[1+1] - \left[\frac{1}{2} - \frac{1}{2}\right]$$

$$= 2\pi$$