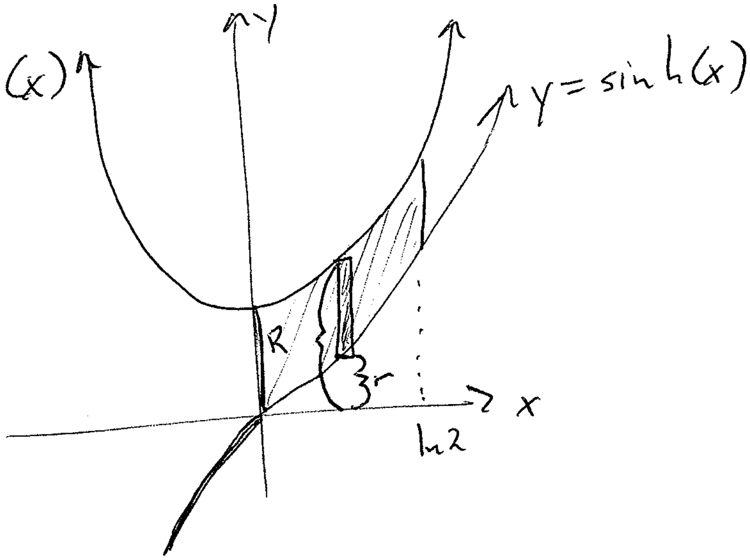
Summer 2007 Midterm #1 Solutions

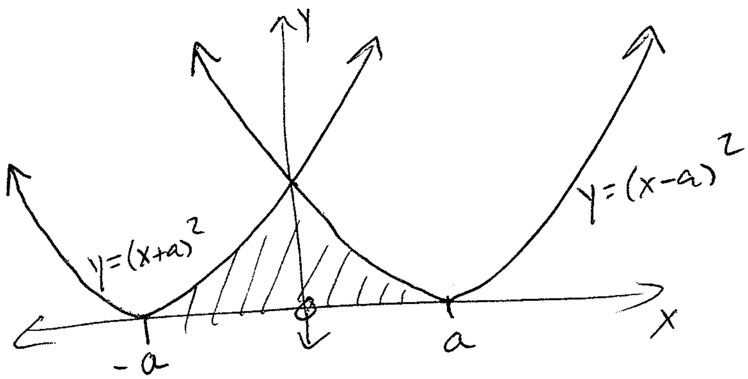
Appm 1360

1.



Washers: $V = \pi \int_0^{\ln(2)} \cosh^2(x) - \sinh^2(x)\,dx = \pi \int_0^{\ln(2)} 1\,dx = \boxed{\pi \ln 2}$

2.



$A = \int_{-a}^{0} (x+a)^2 dx + \int_{0}^{a} (x-a)^2 dx$

$u = x+a$ $\quad$ $v = x-a$

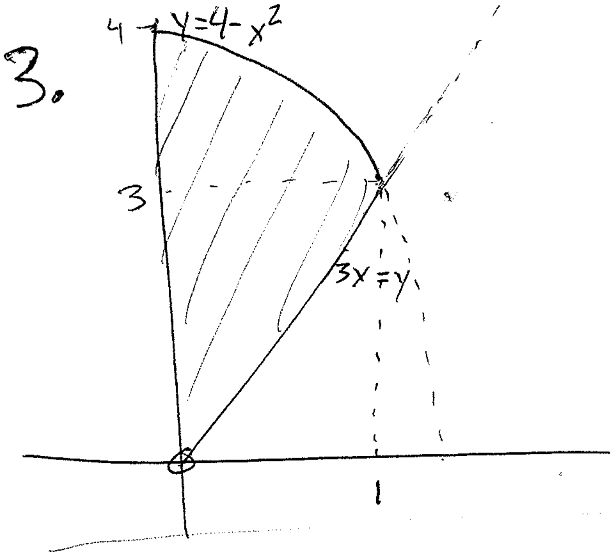
$du = dx$ $\quad$ $dv = dx$

$A = \int_0^a u^2 du + \int_{-a}^{0} v^2 dv$

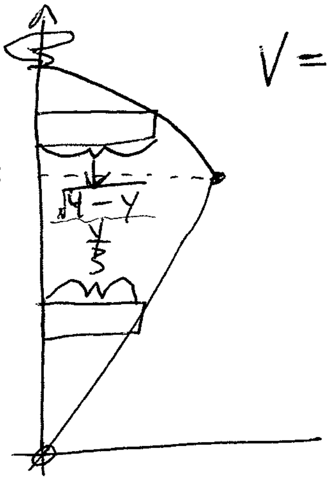
$= \frac{u^3}{3}\Big|_0^a + \frac{v^3}{3}\Big|_{-a}^{0}$

$= \frac{a^3}{3} + \frac{a^3}{3} = \boxed{\frac{2a^3}{3}}$

3.

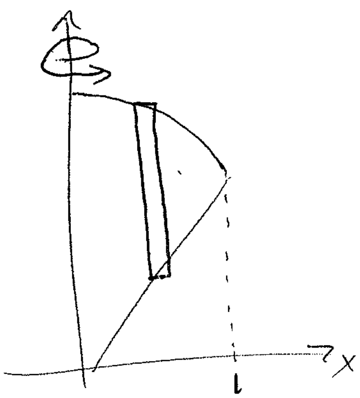


a)



$$V = \pi\int_0^3 \left(\frac{y}{3}\right)^2 dy + \pi\int_3^4 (\sqrt{4-y})^2 dy$$

b)

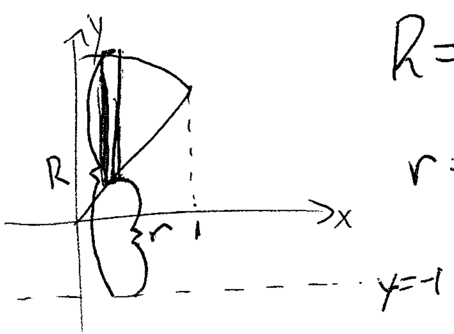


$$V = 2\pi\int rh\,dx = 2\pi\int_0^1 x(4-x^2-3x)\,dx$$

$$= 2\pi\int_0^1 4x-x^3-3x^2\,dx = 2\pi\left[2x^2-\frac{x^4}{4}-x^3\right]_0^1$$

$$= 2\pi\left[2-\frac{1}{4}-1\right] = 2\pi\left[\frac{3}{4}\right] = \boxed{\frac{3\pi}{2}}$$

c)



$R = 4-x^2+1$

$r = 3x+1$

$$V = \pi\int_0^1 (5-x^2)^2-(3x+1)^2\,dx$$

d) (a): $\pi\int_0^3\left(\frac{y}{3}\right)^2 dy + \pi\int_3^4(\sqrt{4-y})^2 dy = \pi\int_0^3 \frac{y^2}{9}\,dy + \pi\int_3^4 4-y\,dy$

$$= \frac{\pi}{27}\left[y^3\right]_0^3 + \pi\left[4y-\frac{y^2}{2}\right]_3^4 = \frac{\pi}{27}\left[27-0\right] + \pi\left[16-8-12+\frac{9}{2}\right]$$

$$= \pi + \frac{\pi}{2} = \boxed{\frac{3\pi}{2}}$$ part (b) above

4. $y = \sqrt{2x+3}$

$y' = \cancel{} = \dfrac{2}{2\sqrt{2x+3}}$

$(y')^2 = \dfrac{1}{2x+3}$

$$SA = 2\pi \int_0^{5/2} y(x)\sqrt{1+(y'(x))^2}\,dx$$

$$= 2\pi \int_0^{5/2} \sqrt{2x+3}\sqrt{1+\frac{1}{2x+3}}\,dx$$

$$= 2\pi \int_0^{5/2} \sqrt{(2x+3) + \frac{2x+3}{2x+3}}\,dx$$

$$= 2\pi \int_0^{5/2} \sqrt{2x+4}\,dx$$

$u = 2x+4$

$du = 2dx$

$\frac{du}{2} = dx$

$$\to = 2\pi \int_4^9 \sqrt{u}\left(\frac{du}{2}\right)$$

$$= 2\pi\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left[u^{3/2}\right]_4^9$$

$$= \frac{2\pi}{3}\left[27-8\right] = \boxed{\frac{38\pi}{3}}$$

5. $y = \int_0^x \operatorname{csch}(t)\,dt$

$y' = \operatorname{csch}(x)$

$$L = \int_{\ln(2)}^{\ln(3)} \sqrt{1+(y')^2}\,dx = \int_{\ln(2)}^{\ln(3)} \sqrt{1+\operatorname{csch}^2(x)}\,dx$$

$$L = \int_{\ln(2)}^{\ln(3)} \sqrt{\coth^2(x)}\,dx = \int_{\ln(2)}^{\ln(3)} \coth(x)\,dx$$

$$L = \int_{\ln(2)}^{\ln(3)} \frac{\cosh(x)}{\sinh(x)}\,dx = \int_{x=\ln(2)}^{x=\ln(3)} \frac{du}{u} = \Big[\ln|u|\Big]_{x=\ln(2)}^{x=\ln(3)}$$

$u = \sinh(x)$

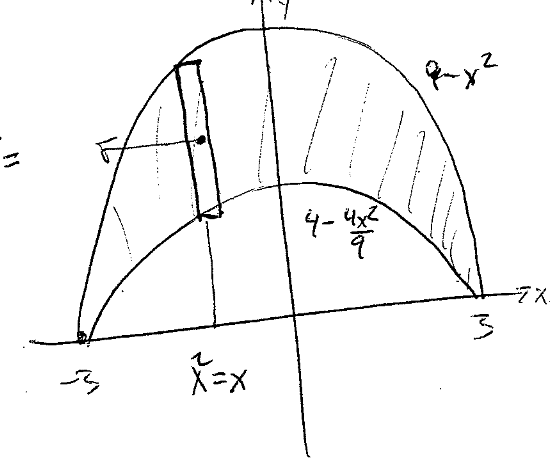
$du = \cosh(x)\,dx$

$$L = \boxed{\ln|\sinh(\ln(3))| - \ln|\sinh(\ln(2))|}$$

$$= \ln\left|\frac{3-\frac{1}{3}}{2}\right| - \ln\left|\frac{2-\frac{1}{2}}{2}\right| = \ln\left|\frac{4}{3}\right| - \ln\left|\frac{3}{4}\right|$$

$$= \ln\left|\frac{4}{3}\Big/\frac{3}{4}\right| = \ln\left|\frac{16}{9}\right| = 2\ln\left|\frac{4}{3}\right|$$

6.



$\tilde{x} = x$

$\tilde{y} = \dfrac{(9-x^2)+(4-\frac{4x^2}{9})}{2} = \dfrac{13-\frac{13x^2}{9}}{2}$

$dm = \delta((9-x^2)-(4-\frac{4x^2}{9}))\,dx$

$= \delta(5-\frac{5x^2}{9})\,dx$

$$M_x = \int \tilde{y}\,dm = \int_{-3}^{3} \left(\frac{13-\frac{13x^2}{9}}{2}\right)\left(\delta\left(5-\frac{5x^2}{9}\right)\right)dx$$

$$M_y = \int \tilde{x}\,dm = \int_{-3}^{3} (x)\left(5-\frac{5x^2}{9}\right)\delta\,dx$$

$$M = \int dm = \int_{-3}^{3} \delta\left(5-\frac{5x^2}{9}\right)dx$$

$\bar{x} = \dfrac{M_y}{M}$

$\bar{y} = \dfrac{M_x}{M}$

by symmetry $\bar{x}=0$: this is seen by noticing that M_y is the integral of an odd function from -3 to 3.