APPM 1360 Exam #2 Summer 2007

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) APPM 1360, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX IN YOUR FINAL ANSWERS. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators, and electronics of any kind are NOT permitted.

1. (20 points) True or False: if the statement is *always* true, put **True** (not just T); if the statement is not *always* true, put **False** (not just F).

   (a) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is defined to be $\lim_{b\to\infty} \int_{-b}^{b} \frac{dx}{1+x^2}$.

   (b) If $a_{n+1} \geq a_n$ for all n, and a_n is bounded below, then $\lim_{n\to\infty} a_n$ exists.

   (c) $\int_{1}^{\infty} \frac{dx}{x(\ln(x))^p}$ converges for $p>1$.

   (d) $\int_{1}^{\infty} \frac{dx}{(x-2)^2}$ converges.

2. (25 points) Determine if the following converge or diverge. Explain your reasoning.

   (a) $\int_{\pi}^{\infty} \frac{2+\cos(x)}{x^{4/3}} dx$ (b) $\int_{-\infty}^{\infty} \frac{1}{\sqrt{x^4+16}} dx$

   (c) $a_n = 8^{1/n}$ (d) $a_n = \left(\frac{n-2}{n}\right)^n$ (e) $a_n = \sin\left(n\pi - \frac{1}{n}\right)$

3. (40 points) Solve the differential equations:

   (a) $x^2 \frac{dy}{dx} = 2y,\ y(1) = e^{-2}$ (b) $\frac{dy}{dx} + y = \frac{1}{1+4e^{2x}}$

   (c) $(t^2 - 3t + 2)\frac{dx}{dt} = 1\ (t>2),\ x(3) = 0$

4. (30 points) Evaluate the following integrals:

   (a) $\int_{\ln(3/4)}^{\ln(4/3)} \frac{e^t\,dt}{(1+e^{2t})^{3/2}}$

   (b) $\int_{0}^{2} \frac{dx}{\sqrt{|x-1|}}$

5. (10 points) It can be shown that $\int_0^T e^{-x^2} dx = \frac{\sqrt{\pi}}{2} - \int_T^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} - \int_T^{\infty} \frac{-2xe^{-x^2}}{-2x} dx.$ Perform integration by parts once on the integral $\int_T^{\infty} \frac{-2xe^{-x^2}}{-2x} dx$ using $u = \frac{1}{2x}$. Assume that for large T the new integral from integration by parts is small enough to throw away. Use this result to estimate the value of $\int_0^T e^{-x^2} dx$ for large T.