

1)

~~2a)~~ a)

$$\begin{aligned}
 V &= \int_0^1 2\pi (\text{shell height}) (\text{shell radius}) dy \\
 &= \int_0^1 2\pi y [1 - (y - y^3)] dy \\
 &= 2\pi \int_0^1 (y - y^2 + y^4) dy = 2\pi \left[ \frac{y^2}{2} - \frac{y^3}{3} + \frac{y^5}{5} \right]_0^1 \\
 &= 2\pi \left( \frac{1}{2} - \frac{1}{3} + \frac{1}{5} \right) = \frac{2\pi}{30} (15 - 10 + 6) = \frac{\pi}{15}
 \end{aligned}$$

b) Washer method

$$\begin{aligned}
 V &= \int_0^1 \pi [R^2(y) - r^2(y)] dy \\
 &= \int_0^1 \pi [1^2 - (y - y^2)^2] dy = \pi \int_0^1 (1 - y^2 - y^6 + 2y^4) dy \\
 &= \pi \left[ y - \frac{y^3}{3} - \frac{y^7}{7} + \frac{2y^5}{5} \right]_0^1 = \pi \left( 1 - \frac{1}{3} - \frac{1}{7} + \frac{2}{5} \right) \\
 &= \frac{97\pi}{105}
 \end{aligned}$$

c) use washer method

$$\begin{aligned}
 V &= \int_0^1 \pi [R^2(y) - r^2(y)] dy \\
 &= \int_0^1 \pi [ [1 - (y - y^3)]^2 - 0 ] dy \\
 &= \pi \int_0^1 [1 - 2(y - y^3) + (y - y^3)^2] dy \\
 \rightarrow &= \pi \int_0^1 (1 + y^6 - 2y + 2y^3 - 2y^4) dy \\
 &= \pi \left[ y + \frac{y^3}{3} + \frac{y^7}{7} - y^2 + \frac{y^4}{2} - \frac{2y^5}{5} \right]_0^1 \\
 &= \frac{121\pi}{210}
 \end{aligned}$$

d) ~~etc~~

$$\begin{aligned}
 V &= \int_0^1 2\pi (h)(r) dy = \int_0^1 2\pi (1-y) [1 - (y - y^3)] dy \\
 &= 2\pi \int_0^1 (1 - 2y + y^2 + y^3) dy = 2\pi \left[ y - y^2 + \frac{y^3}{3} + \frac{y^4}{4} - \frac{y^5}{5} \right]_0^1 \\
 &= \frac{2\pi}{60} (20 + 15 - 12) = \frac{23\pi}{30}
 \end{aligned}$$

b) length of a curve

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx \quad f'(x) = \frac{dy}{dx}$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

c) Surface Area

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$f(x) = x \tan^{-1}(x^2)$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}; \quad |x| \leq 1$$

$$\tan^{-1}(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{2n+1}$$

$$(a^b)^c = a^{bc}$$

$$x \tan^{-1}(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x \cdot x^{4n+2}}{2n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{2n+1}$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} \frac{x^{4(n+1)+3}}{2(n+1)+1}}{(-1)^n \frac{x^{4n+3}}{2n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{4n+7}}{x^{4n+3}} \right| \left| \frac{2n+1}{2(n+1)+1} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{2n+1}{2(n+1)+1} \right| x^4$$

need  $|x^4| < 1$

$$|x|^4 < 1$$

$$|x| < 1$$

$$\boxed{-1 < x < 1}$$

$$f(x) = \frac{x}{\frac{1}{x} + 1} \qquad \frac{1}{1+x} = \sum_{n=0}^{\infty} x^n$$

$$= x \frac{1}{1 - (-\frac{1}{x})} \Rightarrow$$

$$\frac{1}{1 - (-\frac{1}{x})} = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{1}{x})^n}{1}$$

$$\frac{x}{1 + \frac{1}{x}} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{-n} x}{1} = \sum_{n=0}^{\infty} (-1)^n x^{1-n}$$

Radius of convergence

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{(1-(n+1))}}{(-1)^n x^{1-n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{-n}}{x^1 x^{-n}} \right|$$

$$\left| \frac{1}{x} \right| < 1$$

$x \neq 0$  - not defined for  $x=0$

$f(0) = \text{undefined}$

$\sum_{n=0}^{\infty} (-1)^n (\frac{1}{x})^n$  - not defined  $x=0$

Radius of convergence = 0

re-write

$$f(x) = \frac{x^2}{1+x} = x^2 \sum_{n=0}^{\infty} (-1)^n x^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{n+2} \qquad |x| < 1$$

b) True

$$\int_{\frac{2}{\sqrt{3}}}^2 t \sec^{-1}(t) dt$$

$$1) \quad u = \sec^{-1} t$$

$$du = \frac{1}{t\sqrt{t^2-1}} dt$$

$$v = t$$

$$dv = \frac{t^2}{2}$$

$$\int_{\frac{2}{\sqrt{3}}}^2 t \sec^{-1}(t) dt = \left[ \frac{t^2}{2} \sec^{-1}(t) \right]_{\frac{2}{\sqrt{3}}}^2 - \int_{\frac{2}{\sqrt{3}}}^2 \frac{t^2}{2} \left[ \frac{1}{t\sqrt{t^2-1}} \right] dt$$

$$= \left( 2 \cdot \frac{\pi}{3} - \frac{2}{3} \cdot \frac{\pi}{6} \right) - \int_{\frac{2}{\sqrt{3}}}^2 \frac{t}{2\sqrt{t^2-1}} dt$$

$$= \frac{5\pi}{9} - \left[ \frac{1}{2} \sqrt{t^2-1} \right]_{\frac{2}{\sqrt{3}}}^2 \quad \begin{array}{l} \uparrow \\ \text{u-sub} \end{array}$$

$$= \frac{5\pi - 3\sqrt{3}}{9}$$

$$2) \quad \int \frac{6}{(9t^2+1)^2} dt \Rightarrow t = \frac{1}{3} \tan(\theta) \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$dt = \frac{1}{3} \sec^2 \theta d\theta$$

$$9t^2+1 = \sec^2 \theta$$

$$\int \frac{6}{(9t^2+1)^2} dt = \int \frac{6 \left( \frac{1}{3} \sec^2 \theta \right)}{\sec^4(\theta)} d\theta$$

$$= 2 \int \cos^2 \theta d\theta = \theta + \sin \theta \cos \theta + C$$

$$= \theta + \tan \theta \cos^2 \theta + C$$

$$= \tan^{-1}(3t) + \frac{3t}{(9t^2+1)} + C$$

$$\int \frac{-2t^2 + 4t + 6}{t^3 + 2t^2 + t + 2} dt$$

~~$$\frac{-2t^2 + 4t + 6}{t^3 + 2t^2 + t + 2} = \frac{A}{(t^2 + 1)} + \frac{B}{t + 2} = A(t + 2) + B(t^2 + 1)$$~~

~~$$-2t^2 + 4t + 6 = At + 2A + Bt^2 + B$$~~

~~$$t^2: -2 = B$$~~

~~$$t: 4 = A$$~~

~~$$t^0: 6 = 2A + B = 2 \cdot 4 - 2 = 6$$~~

Answer is correct  
Method is not

$$\Rightarrow \int \frac{4}{t^2 + 1} dt + \int \frac{-2}{t + 2} dt$$

$$= 4 \tan^{-1}(t) + C - 2 \ln(t + 2)$$

$$\int e^\theta \sin(\theta) d\theta \quad \text{let } \begin{matrix} dv = e^\theta d\theta \\ v = e^\theta \end{matrix}$$

$$I = e^\theta \sin \theta - \int e^\theta \cos \theta d\theta \quad \begin{matrix} u = \sin \theta \\ du = \cos \theta d\theta \end{matrix}$$

$$I = e^\theta \sin \theta - [e^\theta \cos \theta - \int e^\theta (-\sin \theta) d\theta] \quad \begin{matrix} \text{let } dv = e^\theta d\theta \\ v = e^\theta \\ u = \cos \theta \\ du = -\sin \theta d\theta \end{matrix}$$

$$I = e^\theta \sin \theta - e^\theta \cos \theta - \int e^\theta \sin \theta d\theta$$

$$I = e^\theta (\sin \theta - \cos \theta) - I$$

$$2I = e^\theta (\sin \theta - \cos \theta)$$

$$I = \frac{e^\theta}{2} (\sin \theta - \cos \theta)$$

$$\int \frac{5-x}{x^2-1} dx$$

$$\frac{5-x}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$$5-x = Ax - A + Bx + B$$

$$5 = B - A \quad \Rightarrow \quad B = 5 + A$$

$$-1 = A + B$$

$$-1 = A + 5 + A$$

$$-1 = 2A + 5$$

$$-6 = 2A$$

$$\boxed{-3 = A}$$

$$5 = B + 3$$

$$2 = B$$

$$\int \frac{5-x}{x^2-1} dx = \int \frac{-3}{x+1} + \frac{2}{x-1} dx$$

$$= -3 \ln(x+1) + 2 \ln(x-1) + C$$

$$\int \frac{(1-r^2)^{5/2}}{r^8} dr$$

$$\text{let } r = \sin \theta$$

$$-\pi/2 < \theta < \pi/2$$

$$\int \frac{\cos^5 \theta \cos \theta}{\sin^8 \theta} d\theta = \int \cot^6 \theta \csc^2 \theta d\theta$$

$$= -\frac{\cot^7 \theta}{7} + C = -\frac{1}{7} \left[ \frac{\sqrt{1-r^2}}{r} \right]^7 + C$$

3/a)

f)  $f(x) = \ln(1 + \sqrt{x})$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \quad |x| < 1$$

$$f(x) = \ln(1 + \sqrt{x}) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x^{\frac{n}{2}})}{n}$$

Radius of convergence

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^n (x^{\frac{(n+1)}{2}})}{n+1} \cdot \frac{n}{(-1)^{n-1} x^{\frac{n}{2}}} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{\frac{n}{2}} x^{\frac{1}{2}}}{x^{\frac{n}{2}}} \right| \left| \frac{n}{n+1} \right|$$

$$\Leftrightarrow |x^{\frac{1}{2}}| < 1$$

$$0 < x^{\frac{1}{2}} < 1$$

$$0 < x < 1$$

-check endpoints

$$0 \rightarrow f(0) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1} (0)}{n} = 0$$

-converges absolutely

$$1 \rightarrow f(1) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n}$$

⊗ converges conditionally  
-Alternating series

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

Absolutely converges  
 $0 \leq x < 1$

converges conditionally  
 $x=1$

$$i) a) \begin{aligned} x &= r \cos \theta & = r^2 &= x^2 + y^2 \\ y &= r \sin \theta & \tan \theta &= \frac{y}{x} \end{aligned}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r = \theta = \rho$$

$$\sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$$

ii) When we cross x-axis  $y=0$

$$y = r \sin \theta = \theta \cdot \sin \theta = 0 \quad \left\{ \begin{array}{l} \theta = 0 \\ \sin \theta = 0 \end{array} \right.$$

$$\theta = 2n\pi \quad n=0, 1, 2, \dots$$

- positive x-axis

$$P_1(0,0), P_2(2\pi,0), P_3(4\pi,0)$$

$$c) \text{ Area} = \int_0^{2\pi} \frac{1}{2} r^2 d\theta = \int_0^{2\pi} \frac{1}{2} \theta^2 d\theta$$

$$= \frac{1}{2} \frac{\theta^3}{3} \Big|_0^{2\pi} = \frac{4\pi^3}{3}$$

$$d) \text{ length} = \int_0^{4\pi} \sqrt{r^2 + (r')^2} d\theta = \int_0^{4\pi} \sqrt{1 + 1} d\theta$$

$$r = \theta = \rho \quad r' = 1$$

Handwritten notes on a page with a vertical margin line on the left and horizontal lines. The text is extremely faint and illegible, appearing as light grey smudges and ghosting of characters. Some faint shapes suggest the presence of words or mathematical symbols, but they cannot be transcribed accurately.