(1) $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

let $u = \sqrt{t}$

$du = \frac{1}{2}\frac{1}{\sqrt{t}}\,dt$

$\sqrt{t} \cdot 2\,du = dt$

$\cancel{2}$ $\int \frac{e^{u}}{\sqrt{t}}\sqrt{t}\cdot 2\,du\Big|_{u=\sqrt{t}} = 2\int e^{u}\,du\Big|_{u=\sqrt{t}} = 2e^{\sqrt{t}}+C$

(2) $\cancel{3}$ $\int \frac{\ln(x)}{x+4x\ln^2(x)}\,dx = \int \frac{\ln(x)}{x\,(1+4\ln^2(x))}\,dx$

let $u = 1+4\ln^2(x)$

$du = 4\cdot 2\cdot \ln(x)\cdot \frac{1}{x}\,dx$

$\frac{1}{8}\frac{1}{\ln(x)}\cdot x\;du = dx$

$= \int \frac{\cancel{\ln(x)}}{\cancel{x}\,(u)}\cdot\frac{1}{8}\cdot\frac{1}{\cancel{\ln(x)}}\cdot \cancel{x}\;du\Big|_{u=1+4\ln^2(x)}$

$= \frac{1}{8}\int \frac{1}{u}\,du\Big|_{u=1+4\ln^2(x)} = \boxed{\frac{1}{8}\ln\left[1+4\ln^2(x)\right]+C}$

3 $\cancel{4}$ $\int_{-1}^{3} \frac{4x^2-7}{2x+3}\,dx$

$$\begin{array}{r} 2x-3 \\ 2x+3\;\overline{)\,4x^2-7} \\ -\,\cancel{2}4x^2-6x \\ \hline -6x-7 \\ 6x+9 \\ \hline 2 \end{array}$$

$\int_{-1}^{3} 2x-3+\frac{2}{2x+3}\,dx$

let $u = 2x+3$

$du = 2\,dx$

check

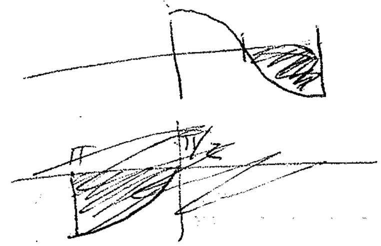
$4x^2+6x-6x-9+2$

$4x^2-7$

$\frac{x^2}{2}\Big|_{-1}^{3} - 3x\Big|_{-1}^{3} + \ln(2x+3)\Big|_{-1}^{3}$

$\frac{9}{2}-\frac{1}{2}-3[3+1]+\ln(9)-\cancel{\ln(1)}$

$= \frac{8}{2}-12+\ln(9) = -8+\ln(9)$



4) $\int_{\pi/2}^{\pi} \sqrt{1-\sin^2(\theta)}\, d\theta$

$= \int_{\pi/2}^{\pi} \sqrt{\cos^2(\theta)}\, d\theta = \int_{\pi/2}^{\pi} \cos(\theta)\, d\theta$

$= \sin\theta \Big|_{\pi/2}^{\pi} = \sin(\pi) - \sin(\pi/2) = -1$

5) $\int_1^e x^3 \ln(x)\, dx$

$\cancel{x^3}$, $\cancel{3x^2}$, $\cancel{6x}$, $\cancel{6}$, $\cancel{0}$

let $dv = x^3$

$v = \frac{x^4}{4}$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$\frac{x^4}{4} \ln(x) \Big|_1^e - \int_1^e \frac{x^4}{4} \cdot \frac{1}{x} dx$

$\frac{e^4}{4} \ln(e) - \left[\frac{1}{4} \cancel{\ln(1)}\right]$ 0

$- \int_1^e \frac{x^3}{4} \cdot dx$

$\frac{e^4}{4} - \frac{x^4}{4 \cdot 4} \Big|_1^e = \frac{e^4}{4} - \left[\frac{e^4}{16} - \frac{1}{16}\right]$

$= \frac{4e^4}{16} - \frac{e^4}{16} - \frac{1}{16} = \frac{1}{16}\left[3e^4 - 1\right]$

(6) $I = \int e^x \sin(x)\,dx$ $\qquad dv = e^x\,dx$

$v = e^x$

$u = \sin(x)$

$I = e^x \sin x - \int e^x \cos(x)\,dx \qquad du = \cos(x)\,dx$

let $dv = e^x\,dx$

$I = e^x \sin x - e^x \cos x + \int e^x(-)\sin x\,dx \qquad v = e^x$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$2I = e^x \sin x - e^x \cos x$

$I = e^x \left[\frac{\sin x - \cos x}{2} \right]$

(7)

$$⑦ \quad I = \int_1^{\sqrt{3}} \frac{3t^2+t+4}{t^3+t}\,dt$$

$$\frac{3t^2+t+4}{t^3+t} = \frac{3t^2+t+4}{t(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{t^2+1}$$

$$3t^2+t+4 = (t^2+1)A + (Bt+C)t$$

$$\Rightarrow \cancel{3t^2 = At^2}$$

$$\cancel{t+4 = A+Bt}$$

$$3t^2+t+4 = At^2 + A + Bt^2 + Ct$$

$$3t^2 = (A+B)t^2$$

$$t = Ct \quad \Rightarrow C = 1$$

$$4 = A$$

$$3 = A+B$$

$$3 = 4+B \quad \Rightarrow B = -1$$

$$\Rightarrow \frac{3t^2+t+4}{t(t^2+1)} = \frac{4}{t} + \frac{(-t+1)}{t^2+1}$$

$$\Rightarrow I = \int_1^{\sqrt{3}} \frac{4}{t} + \frac{(-t+1)}{t^2+1}\,dt$$

$$I = 4\ln(t)\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} \frac{t}{t^2+1}\,dt + \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt$$

$$= 4\ln(\sqrt{3}) - \cancelto{0}{4\ln(1)} - \frac{1}{2}\ln(t^2+1)\Big|_1^{\sqrt{3}} + \tan^{-1} t\Big|_1^{\sqrt{3}}$$

$$= \ln(9) - \ln(\sqrt{2}) + \frac{\pi}{12}$$

$$= 2\ln(3) - \ln(2) + \pi/3 + \frac{1}{2}\ln(2) + \pi/4 = \ln\left(\frac{9}{\sqrt{2}}\right) + \frac{\pi}{12}$$

(8) $\displaystyle\int \frac{s^4+81}{s(s^2+9)^2}\,ds$

$$\frac{s^4+81}{s(s^2+9)^2} = \frac{A}{s} + \frac{Bs+C}{s^2+9} + \frac{Ds+E}{(s^2+9)^2}$$

$$\Rightarrow s^4+81 = A(s^2+9)^2 + (Bs+C)s(s^2+9)$$
$$(Ds+E)s$$

$$= As^4 + A18s^2 + A81$$
$$+ Bs^4 + Cs^3 + 9Bs^2 + 9Cs + Ds^2 + Es$$

$$= (A+B)s^4 + Cs^3 + (18A+9B+D)s^2$$
$$(9C+E)s + 81A$$

$\Rightarrow 81A = 81 \Rightarrow A = 1$

$A+B = 1 \Rightarrow B = 0$

$C = 0$ ← no matching s^3 on LHS

$D = -18$

$$\Rightarrow \int \frac{s^4+81}{s(s^2+9)^2}\,ds = \int \frac{1}{s}\,ds - 18\int \frac{s\,ds}{(s^2+9)^2}$$

$$= \ln|s| + \frac{9}{(s^2+9)} + C$$

9) $\displaystyle\int_0^{\ln(4)} \frac{e^t}{\sqrt{e^{2t}+9}}\,dt$ let $u = e^{2t}$

$du = 2e^{2t}\,dt$

9) $\displaystyle\int_0^{\ln(4)} \frac{e^t}{\sqrt{e^{2t}+9}}\, dt$

Let $e^t = 3\tan\theta$

$t = \ln(3\tan\theta)$

$dt = \dfrac{1}{3\tan(\theta)} \sec^2\theta\, d\theta$

$t = \ln(4) \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$t = 0 \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$

$$\int_{\tan^{-1}(1/3)}^{\tan^{-1}\left(\frac{4}{3}\right)} \frac{3\tan(\theta)\sec^2(\theta)\, d\theta}{\tan(\theta)\, 3\sec(\theta)}$$

$$= \int_{\tan^{-1}\left(\frac{1}{3}\right)}^{\tan^{-1}\left(\frac{4}{3}\right)} \sec\theta\, d\theta = \Big[\ln[\sec(\theta)+\tan(\theta)]\Big]_{\tan^{-1}(1/3)}^{\tan^{-1}(4/3)}$$

$$= \ln\left(\frac{5}{3} + \frac{4}{3}\right) - \ln\left(\frac{\sqrt{10}}{3} + \frac{1}{3}\right)$$

$$= \ln(9) - \ln(1+\sqrt{10})$$

10) $\sqrt{x^2-9}\, \dfrac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2-9}}$$

$\displaystyle\int dy = \int \frac{1}{\sqrt{x^2-9}}\, dx$ Let $x = 3\sec\theta$ $\theta \in$ [illegible]

$dx = 3\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-9} = 3\tan\theta$

$$y = \int \frac{3\sec(\theta)\tan\theta\, d\theta}{\tan\theta}$$

$$= \int \sec\theta\, d\theta = \ln[\sec\theta + \tan\theta]| + C$$

$y = \ln\left|\dfrac{x}{3} + \dfrac{\sqrt{x^2-9}}{3}\right| + C$ $x = 5$ + $y = \ln(3)$

$\Rightarrow \quad \ln(3) = \ln(3) + C \Rightarrow C = 0$

$$y(x) = \ln \left| \frac{x}{3} + \frac{\sqrt{x^2+9}}{3} \right|$$