APPM 1360 EXAM #1 September 26, 2007

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor (Dougherty, section 10 or Li, section 20 with lecture at 1 pm or section 30 with lecture at 2 pm). This exam is worth 100 points and has 5 questions. A list of formulas is given on the back of this exam. **Show all work!** Answers with no justification will receive no points.

1. (20 points) Find the requested information.
   (a) Find dy/dx for $y = \ln(\cosh x))$.
   (b) $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx$
   (c) Find the area between the curve $y = \dfrac{2\ln x}{x}$ and the x-axis from $x = 1$ to $x = e$. Simplify your answer.
2. (15 points) Solve (e.g. find an explicit formula for y in terms of x) the initial value problem given by $\dfrac{dy}{dx} = e^x\,(y^2+4)$ with $y(0) = 2$.
3. (15 points) A solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes are vertical squares whose base edges run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$. Find the volume of the solid.
4. (35 points) Consider the region bounded by the two curves $y = x^2$ and $y = 3x^2 - 2$. Set up, but do **not** evaluate, the integrals required to find each of the following quantities.
   (a) The center of mass, assuming the thin, flat plate has constant density of $3g/cm^2$.
   (b) The volume of the solid formed by revolving the area bounded by $y = x^2$, $y = 3x^2 - 2$, and $x \geq 0$ about the y-axis.
   (c) The volume of the solid formed by revolving the area bounded by $y = x^2$, $y = 3x^2 - 2$, and $x \geq 0$ about the line $y = 2$.
5. (15 points) Find the length of the segment of the curve $y = (1/2)\cosh(2x)$ from $x = 0$ to $x = \ln\sqrt{5}$. Simplify your answer as much as possible.

Extra Credit (5 points): Evaluate the following limit by identifying it with a definite integral:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k}$$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name and section, and a grading key.

APPM 1360 — Exam #1 Formula Sheet — September 26, 2007

1. A short table of integrals. In the following, $a \neq 0$.

   (a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

   (b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

   (c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

   (d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

   (e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

   (f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

   (g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\operatorname{sech}^{-1}(u/a) + C$ for $0 < u < a$

   (h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\operatorname{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. Some identities.

   (a) $\sin^2 x + cos^2 x = 1$

   (b) $\sin^2 x = (1 - \cos(2x))/2$

   (c) $\cos^2 x = (1 + \cos(2x))/2$

   (d) $\cosh^2 x - \sinh^2 x = 1$