1 (a) $y = \ln(\cosh x) \qquad \frac{dy}{dx} = \frac{\sinh x}{\cosh x}$

(b) $\int \frac{1}{x+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx \qquad$ Let $u = \sqrt{x}+1$, $du = \frac{1}{2}x^{-1/2}\,dx$

$= \int \frac{2}{u}\,du = 2\ln(\sqrt{x}+1) + C$

(c) Area $= \int_1^e \frac{2\ln x}{x}\,dx \qquad$ Let $u = \ln x$, $du = \frac{1}{x}\,dx$

$= \int_0^1 2u\,du = 2\frac{u^2}{2}\Big|_0^1 = \boxed{1}$

2. $\frac{dy}{dx} = e^x(y^2+4) \qquad y(0) = 2$

$\int \frac{dy}{y^2+4} = \int e^x\,dx$

$\frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) = e^x + C$

$y(0)=2 \Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right) = e^0 + C \Rightarrow \frac{1}{2}\cdot\frac{\pi}{4} = 1 + C \Rightarrow C = \frac{\pi}{8} - 1$

$\therefore \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) = e^x + \frac{\pi}{8} - 1$

$\Rightarrow \tan^{-1}\left(\frac{y}{2}\right) = 2e^x + \frac{\pi}{4} - 2$

$\Rightarrow \frac{y}{2} = \tan\left(2e^x + \frac{\pi}{4} - 2\right)$

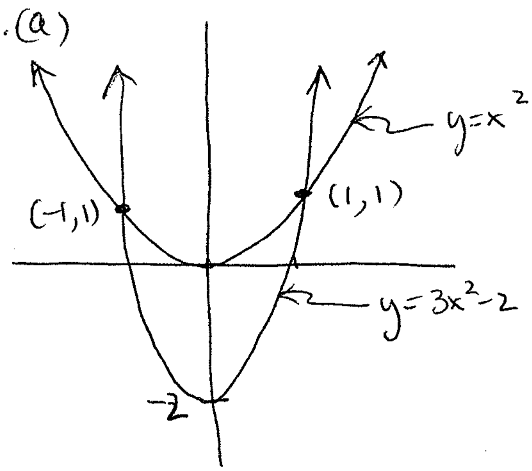
$\Rightarrow y(x) = 2\tan\left(2e^x + \frac{\pi}{4} - 2\right)$

3. length of one side of square $= \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right) = 2\sqrt{1-x^2}$

$A(x) = \left(2\sqrt{1-x^2}\right)^2 = 4(1-x^2)$

$\therefore V(x) = \int_{-1}^1 4(1-x^2)\,dx = 4\left[x - \frac{x^3}{3}\right]_{-1}^1 = 4\left(\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right) = \boxed{\frac{16}{3}}$

4.(a)



$\bar{x} = 0$ by symmetry

$dA = \left(x^2 - (3x^2-2)\right)dx = (2-2x^2)\,dx$

$dm = \delta\,dA = 3(2-2x^2)\,dx$

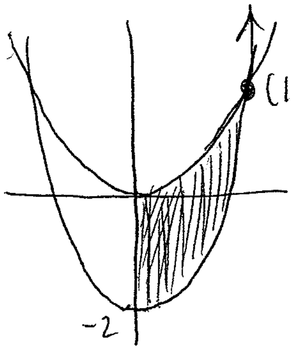
$(\tilde{x},\tilde{y}) = \left(x, \frac{x^2+3x^2-2}{2}\right) = (x, 2x^2-1)$

$\therefore$ Mass $M = \int dm = \int_{-1}^1 3(2-2x^2)\,dx$

$M_x = \int \tilde{y}\,dm = \int_{-1}^1 (2x^2-1)\,3(2-2x^2)\,dx$

$\bar{y} = \frac{M_x}{M} = \frac{\int_{-1}^1 (2x^2-1)(6-6x^2)\,dx}{\int_{-1}^1 (6-6x^2)\,dx}$

4 (b) area bounded by $y=x^2$, $y=3x^2-2$ and $x \geq 0$ (shaded area)



Rotate about y-axis.

shells: $V = \int_0^1 2\pi \text{(shell radius)(shell height)}\, dx$

$$= \int_0^1 2\pi x (2-2x^2)\, dx$$

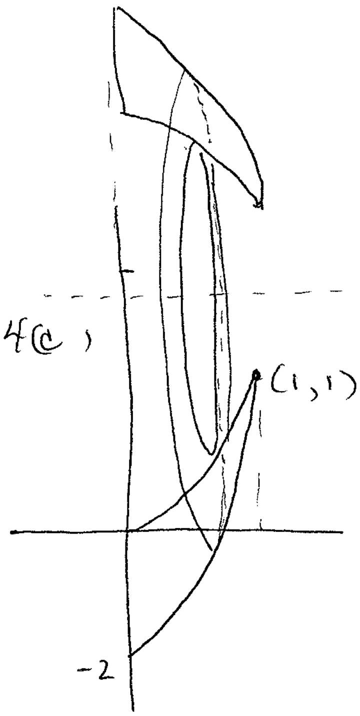
(Note: Can use disks + washers, but it is much more difficult. Need to solve for x in terms of y:

$y = 3x^2-2 \Rightarrow x = \sqrt{\frac{y+2}{3}}$ (since $x \geq 0$)

$y = x^2 \Rightarrow x = \sqrt{y}$

Then $V = \pi \int_{-2}^{0} \underbrace{\left(\sqrt{\frac{y+2}{3}}\right)^2}_{(\text{radius})^2} dy + \pi \int_0^1 \underbrace{\left(\sqrt{\frac{y+2}{3}}\right)^2}_{\text{outer radius}^2} - \underbrace{\left(\sqrt{y}\right)^2}_{\text{inner radius}^2} dy$

4(c)



Use washers for volume rotated about line $y=2$.

$$V = \pi \int_0^1 \left[(\text{outer radius})^2 - (\text{inner radius})^2\right] dx$$

$$= \pi \int_0^1 \left[\left(2-(3x^2-2)\right)^2 - (2-x^2)^2\right] dx$$

$$= \pi \int_0^1 \left[(4-3x^2)^2 - (2-x^2)^2\right] dx$$

(Note: Can use shells, but it is more difficult + requires two separate integrals.)

5. $y = \frac{1}{2}\cosh(2x) \Rightarrow \frac{dy}{dx} = \sinh x$

$\therefore L = \int_0^{\ln\sqrt{5}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_0^{\ln\sqrt{5}} \sqrt{1+\sinh^2 x}\, dx = \int_0^{\ln\sqrt{5}} \underbrace{\cosh x}_{\text{don't need absolute values since } \cosh x \geq 0} dx$

$$= \frac{1}{2}\sinh(2x) \Big|_0^{\ln\sqrt{5}}$$

$$= \frac{1}{2}\, \frac{e^{2x}-e^{-2x}}{2} \Big|_0^{\ln\sqrt{5}}$$

$$= \frac{e^{2\ln\sqrt{5}} - e^{-2\ln\sqrt{5}}}{4}$$

$$= \frac{5-\frac{1}{5}}{4} = \boxed{\frac{6}{5}}$$