APPM 1360 EXAM #2 FALL 2007

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor (Dougherty, section 10, or Li, section 30 with lecture at 1 pm or section 20 with lecture at 2 pm). This exam is worth 100 points and has 5 questions. A list of formulas is given on the back of this exam. **Show all work!** Answers with no justification will receive no points.

1. (30 points) Evaluate the following integrals. If an integral is improper, determine whether it converges or diverges. If it converges, evaluate it. If it diverges, justify your answer.

$$(a)\quad \int_1^e x\ln(x)\,dx \qquad (b)\quad \int_0^{1/2} \frac{6}{\sqrt{1-4x^2}}\,dx \qquad (c)\quad \int_{-1}^{1} \frac{1}{t^3}\,dt$$

2. (20 points) Determine whether the following sequences converge or diverge. If the sequence converges, find its limit.

$$(a)\quad a_n = 1+(-1)^n \qquad (b)\quad b_n = \frac{1}{\sqrt{n^2+n}-\sqrt{n^2+1}} \text{ for } n \geq 2 \qquad (c)\quad c_n = n\ln\left(1+\frac{2}{n}\right)$$

3. (15 points) Consider the sequence $a_n = \dfrac{3}{n(n+1)}$ and the series $\sum_{n=1}^{\infty} a_n$.

   (a) Does the sequence $\{a_n\}$ converge? If so, what is its limit?

   (b) Find a simple expression for the n^{th} partial sum S_n.

   (c) Does the *sequence* $\{S_n\}$ converge? If so, what is its limit?

   (d) Does the *series* $\sum_{n=1}^{\infty} a_n$ converge? If so, what is its limit?

4. (10 points) For each of the following, state whether the claim is true or false. If the claim is true, justify your answer using known theorems or techniques. If the statement is false, provide a counterexample.

   (a) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of nonnegative terms and if $\sum_{n=1}^{\infty}(2a_n+3b_n)$ converges then both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ must converge.

   (b) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of nonnegative terms and if $\sum_{n=1}^{\infty}(2a_n+3b_n)$ diverges then both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ must diverge.

5. (25 points) Determine whether the following series converge or diverge. Justify your answer and be sure to name any test you use.

$$(a)\quad \sum_{n=2}^{\infty} \frac{1}{n\ln(n^3)} \qquad (b)\quad \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{5^n} \qquad (c)\quad \sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name (Dougherty or Li), and a grading key.

1. A short table of integrals. In the following, $a \neq 0$.

   (a) $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

   (b) $\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

   (c) $\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

   (d) $\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

   (e) $\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

   (f) $\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

   (g) $\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\mathrm{sech}^{-1}(u/a) + C$ for $0 < u < a$

   (h) $\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\mathrm{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. Some identities.

   (a) $\sin^2 x + cos^2 x = 1$

   (b) $\sin^2 x = (1 - \cos(2x))/2$

   (c) $\cos^2 x = (1 + \cos(2x))/2$

   (d) $\cosh^2 x - \sinh^2 x = 1$