APPM 1360 EXAM #3 FALL 2007

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor (Dougherty, section 10, or Li, section 30 with lecture at 1 pm or section 20 with lecture at 2 pm). This exam is worth 100 points and has 5 questions. **Show all work!** Answers with no justification will receive no points.

1. (20 points) A few unrelated questions. Justify your answer in each case.
   (a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$ converge or diverge?
   (b) Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converge or diverge?
   (c) Find the limit: $\displaystyle\lim_{x\to 0} \frac{\sin x - x + (x^3/6)}{x^5}$.

2. (30 points) Consider the series given by $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$ Justify each answer carefully and completely.
   (a) What is the radius of convergence for this series?
   (b) For what values of x does the power series converge absolutely?
   (c) For what values of x does the power series converge conditionally (but not absolutely)?
   (d) For what values of x does the power series diverge?

3. (15 points) In each of the following parts, you will have a function equal to a series on the interval $-1 < x < 1$.
   (a) Find the geometric series which is equal to $\dfrac{1}{1-x}$.
   (b) Differentiate the geometric series from part (a) to find a new series equal to the derivative of $\dfrac{1}{1-x}$.
   (c) Multiply both the function and the series in part (b) by x.
   (d) Substitute an appropriate value of x into the result of part (c) to evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$.

4. (26 points) Let $f(t) = \cos(t^2)$. (Hint: You may use your knowledge of the Maclaurin series for $\cos(t)$ to answer the following questions.)
   (a) Find the Maclaurin series for $f(t) = \cos(t^2)$.
   (b) Use the first two nonzero terms of your series from part (a) to estimate $\displaystyle\int_0^1 \cos(t^2)\, dt$
   (c) Estimate the error being made in part (b). Leave your answer in fractional form.
   (d) Does your estimate in part (b) overestimate or underestimate the exact value of $\displaystyle\int_0^1 \cos(t^2)\, dt$? Explain.

5. (9 points) Classify the equation (e.g. circle, ellipse, hyperbola, etc.) for the different values of c given below. (For this problem only you do not need to show your work.)

$$\frac{x^2}{c-4} + \frac{y^2}{10-c} = 1$$

$$(a) \quad c < 4 \qquad (b) \quad 4 < c < 10 \qquad (c) \quad 10 < c$$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name (Dougherty or Li), and a grading key.