**APPM 1360** **FINAL EXAM** **FALL 2007**

**On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor (Dougherty, section 10, or Li, section 30 with lecture at 1 pm or section 20 with lecture at 2 pm).** This exam is worth 200 points and has 7 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page.

1. (40 points) Evaluate the following integrals and limits. If the integral or limit diverges, explain why.

$$(a)\ \int_0^{\frac{\pi}{2}} \cos(x)\sqrt{1+\sin(x)}\,dx \qquad (b)\ \int_0^2 \frac{1}{(t-1)^5}\,dt \qquad (c)\ \int \frac{1}{x^2-3x}\,dx$$

$$(d)\ \lim_{x\to 0^+} \frac{\cos(\sqrt{x}) - 1 + \frac{x}{2}}{x^2} \qquad (e)\ \lim_{n\to\infty} (1-\frac{x}{n})^n$$

2. (25 points) Find the $x$-coordinate of the center of mass of a thin plate of constant density covering the region enclosed by the curve $y = \ln x$ and the $x$-axis for $1 \le x \le e$.

3. (25 points) For this problem, consider the curve $y = x^3$ on the interval $0 \le x \le 2$. Set up, but do not evaluate, the integrals to find the requested quantities.
   (a) The surface area generated by revolving the curve around the x-axis.
   (b) The volume of the solid formed by revolving the area bounded by the $x-$axis, the curve and the line $x = 2$ around the $y-$axis.
   (c) The volume of the solid formed by revolving the same region as part (b) around the line $y = 10$.

4. (27 points) Determine whether the following series converge or diverge. Justify each answer and be sure to name any test you use.

$$(a)\ \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2} \qquad (b)\ \sum_{n=1}^{\infty} \frac{3+\cos(n)}{3^n} \qquad (c)\ \sum_{n=1}^{\infty} \frac{n!\,5^n}{(2n+1)!\,3^n}$$

5. (25 points) For what values of $x$ do the following series converge absolutely, converge conditionally or diverge? Be sure to name any test you use.

$$(a)\ \sum_{n=1}^{\infty} \left(1-\frac{x}{n}\right)^n \qquad (b)\ \sum_{n=1}^{\infty} \frac{x^n}{2^n n(n+2)}$$

6. (23 points) Recall that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.
   (a) Find the Maclaurin series for $\cosh(x)$.
   (b) Find the Maclaurin series for $\cosh(x^2)$.
   (c) Use the first two nonzero terms of your series from part (b) to estimate $\int_0^1 \cosh(x^2)\,dx$
   (d) (Extra credit: 10 pts) Estimate the error being made in part (c).

7. (35 points)
   (a) Find the length of the curve given by $x = t^3$ and $y = 3t^2/2$ over the interval $0 \le t \le \sqrt{3}$.
   (b) Find the area inside the curve $r = 2\cos\theta$ and outside of the curve $r = 1$.

*Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name (Dougherty or Li), and a grading key.*

**APPM 1360 — Final Exam Formula Sheet — December 15, 2007**

1. A short table of integrals. In the following, $a \neq 0$.

   (a) $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

   (b) $\displaystyle\int \frac{du}{a^2 + u^2} = (1/a)\tan^{-1}(u/a) + C$

   (c) $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

   (d) $\displaystyle\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

   (e) $\displaystyle\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

   (f) $\displaystyle\int \frac{du}{a^2 - u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

   (g) $\displaystyle\int \frac{du}{u\sqrt{a^2 - u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

   (h) $\displaystyle\int \frac{du}{u\sqrt{a^2 + u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. Some identities.

   (a) $\sin^2 x + cos^2 x = 1$

   (b) $\sin^2 x = (1 - \cos(2x))/2$

   (c) $\cos^2 x = (1 + \cos(2x))/2$

   (d) $\cosh^2 x - \sinh^2 x = 1$