APPM 1360 Final Exam Formula Sheet

1. A short table of integrals. In the following, $a \neq 0$.

(a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

(b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

(c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

(d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

(e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

(f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

(g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

(h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. Some trig identities.

(a) $\sin^2 x + cos^2 x = 1$

(b) $\cos^2 x = (1+\cos(2x))/2$

(c) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(d) $\sin^2 x = (1-\cos(2x))/2$

(e) $\cosh^2 x - \sinh^2 x = 1$

(f) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

3. Some useful trig substitutions.

$x = a\tan\theta$ replaces $a^2 + x^2$ by $a^2\sec^2\theta$

$x = a\sec\theta$ replaces $x^2 - a^2$ by $a^2\tan^2\theta$

$x = a\sin\theta$ replaces $a^2 - x^2$ by $a^2\cos^2\theta$

4. Some useful limits.

(a) $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n} = 0$

(b) $\displaystyle\lim_{z\to 0} \frac{\sin z}{z} = 1$

(c) $\displaystyle\lim_{n\to\infty} \sqrt[n]{n} = 1$

(d) $\displaystyle\lim_{n\to\infty} \sqrt[n]{\ln(n)} = 1$

(e) $\displaystyle\lim_{n\to\infty} \sqrt[n]{x} = 1$ for $x > 0$

(f) $\displaystyle\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$ for any x

(g) $\displaystyle\lim_{n\to\infty} x^n = 0$ for $|x| < 1$

(h) $\displaystyle\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for any x

(i) $\displaystyle\lim_{n\to\infty} \frac{n!}{n^n} = 0$

Note: in (e) - (h), x remains **fixed** as $n \to \infty$.

5. Frequently used Maclaurin series

(a) $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$

(b) $\displaystyle\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ for $|x| < \infty$

(c) $\displaystyle\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$, for $-1 < x \leq 1$

(d) $\displaystyle(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$ for $|x| < 1$

(e) $\displaystyle e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for $|x| < \infty$

(f) $\displaystyle\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ for $|x| < \infty$

(g) $\displaystyle\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}$, for $|x| \leq 1$

APPM 1360 Final Exam Formula Sheet

1. A short table of integrals. In the following, $a \neq 0$.

   (a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

   (b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

   (c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

   (d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

   (e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

   (f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

   (g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

   (h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$

2. Some trig identities.

   (a) $\sin^2 x + \cos^2 x = 1$
   (b) $\cos^2 x = (1+\cos(2x))/2$
   (c) $\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$
   (d) $\sin^2 x = (1-\cos(2x))/2$
   (e) $\cosh^2 x - \sinh^2 x = 1$
   (f) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

3. Some useful trig substitutions.

   $x = a\tan\theta$ replaces $a^2 + x^2$ by $a^2\sec^2\theta$
   $x = a\sec\theta$ replaces $x^2 - a^2$ by $a^2\tan^2\theta$
   $x = a\sin\theta$ replaces $a^2 - x^2$ by $a^2\cos^2\theta$

4. Some useful limits.

   (a) $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n} = 0$
   (b) $\displaystyle\lim_{z\to 0} \frac{\sin z}{z} = 1$
   (c) $\displaystyle\lim_{n\to\infty} \sqrt[n]{n} = 1$
   (d) $\displaystyle\lim_{n\to\infty} \sqrt[n]{\ln(n)} = 1$
   (e) $\displaystyle\lim_{n\to\infty} \sqrt[n]{x} = 1$ for $x > 0$
   (f) $\displaystyle\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$ for any x
   (g) $\displaystyle\lim_{n\to\infty} x^n = 0$ for $|x| < 1$
   (h) $\displaystyle\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for any x
   (i) $\displaystyle\lim_{n\to\infty} \frac{n!}{n^n} = 0$

   Note: in (e) - (h), x remains **fixed** as $n \to \infty$.

5. Frequently used Maclaurin series

   (a) $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$
   (b) $\displaystyle\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ for $|x| < \infty$
   (c) $\displaystyle\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$, for $-1 < x \leq 1$
   (d) $\displaystyle(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$ for $|x| < 1$
   (e) $\displaystyle e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for $|x| < \infty$
   (f) $\displaystyle\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ for $|x| < \infty$
   (g) $\displaystyle\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}$, for $|x| \leq 1$