

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 5 questions. A list of formulas is given on the back of this exam. **Show all work!** Answers with no justification will receive no points.

1. (25 points) Evaluate the following integrals. If an definite integral is improper, determine whether it converges or diverges. If it converges, evaluate it. If it diverges, justify your answer.

$$(a) \int \frac{1}{\sqrt{9-4t^2}} dt \quad (b) \int_e^\infty \frac{6}{x(\ln x)^2} dx \quad (c) \int_{-2}^2 \frac{1}{t} dt$$

2. (15 points) Determine whether the following sequences converge or diverge. If the sequence converges, find its limit.

$$(a) a_n = (-1)^n \frac{n}{n+1} \quad (b) b_n = \frac{1}{\sqrt{n^2+2n+1} - \sqrt{n^2+n+1}} \quad (c) c_n = n \ln \left(1 + \frac{4}{n}\right)$$

3. (15 points) For each of the following, state whether the claim is true or false. If the statement is false, provide a counterexample.

(a) If both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge then $\sum_{n=1}^{\infty} (2a_n * b_n)$ must converge.

(b) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of nonnegative terms and if $a_n < b_n$ for all n then $\sum_{n=1}^{\infty} a_n$ always converges.

(c) For the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$, if you use the first two terms to estimate the sum of the series, then the magnitude of the error is less than 0.1.

4. (20 points) Determine whether the following series converge or diverge. Justify your answer and be sure to name any test you use.

$$(a) \sum_{n=2}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^n \quad (b) \sum_{n=2}^{\infty} \frac{3}{n \ln n} \quad (c) \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^{1.5}} \quad (d) \sum_{n=1}^{\infty} \frac{5^n}{n!}$$

5. (25 points) Determine whether the following series converge absolutely, converge conditionally, or diverge. Justify your answer and be sure to name any test you use.

$$(a) \sum_{n=2}^{\infty} (-1)^n \frac{\tan^{-1}(n)}{n^2+1} \quad (b) \sum_{n=2}^{\infty} \frac{2(-1)^n}{n \ln n} \quad (c) \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \quad (d) \sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{(2n)!}$$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name (Li, Nelson, Scheels, or Vasil), and a grading key.