

On the front of your bluebook, please write: a grading key, your name, student ID, recitation number and instructor. This exam is worth 100 points. Show all work. Answers with no justification will receive NO points.

1. (24 points) For what values of x do the following series converge absolutely, conditionally or diverge. Justify every answer.

a) $\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$

b) $\sum_{n=1}^{\infty} \frac{(\cos n\pi)}{3^n} x^n$

c) $\sum_{n=1}^{\infty} \frac{(2x-8)^n}{n!}$

2. (20 points)

a) How many terms of the series are needed to approximate $f(x) = \int_0^1 \sin t^3 dt$ with $|E| \leq 10^{-4}$

b) Use series to evaluate $\lim_{x \rightarrow 0} \frac{\cos x^3 - 1 + \frac{x^6}{2}}{2x^{12}}$

3. (24 points)

a) Find the McLaurin series for $\frac{1}{1+x}$ from the function's derivatives. Justify all of your work.

b) Write the first 3 terms of the series for $\int_0^1 e^{x^2} dt$

c) Find the error bound if you approximate 3b with only one term.

4. (8 points) Write the first three terms of the binomial series for $y = (1-3x)^{\frac{1}{2}}$

5. (24 points)

a) For each of parts i, ii and iii below, classify the equation (ie, ellipse, hyperbola, parabola) when you insert each of the given values of "q".

$$\frac{x^2}{q-5} + \frac{y^2}{8-q} = 1$$

i) $q < 5$ ii) $5 < q < 8$ iii) $q > 8$

b) Draw the graph of $x^2 - 4y^2 = 16$, being sure to label axes and important points.

c) Find the vertices, asymptotes, eccentricity and foci for the equation in 5b