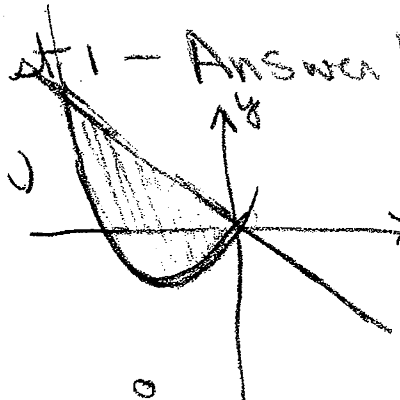
st 1 – Answer Key

1)



b) $\int_{-4}^{0}\left[-x-(4x+x^2)\right]dx = \int_{-4}^{0}(-5x-x^2)\,dx = -\frac{5x^2}{2}-\frac{x^3}{3}\Big|_{-4}^{0}$

$= -\left(\frac{-5(16)}{2} - \frac{-64}{3}\right) = 40 - \frac{64}{3} = \boxed{\frac{56}{3}}$

c)

base $= -x - 4x - x^2 = -5x - x^2$

$-x = 4x + x^2$

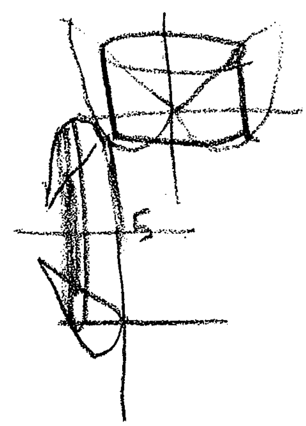
$x^2 + 5x = 0$

$(x)(x+5) = 0$

$A = bh = b^2 = (-5x-x^2)^2$

$\int_{-5}^{0}(-5x-x^2)^2\,dx$

d) Shells: $2\pi\int_{-5}^{0} -x(-x-4x-x^2)\,dx$



e) Washers $\pi\int_{-5}^{0}\left[5-(4x+x^2)\right]^2 - (5+x)^2\,dx$

2a) $\int_{1}^{2}\frac{x-5}{x(x+5)}\,dx = \int_{1}^{2}\left(\frac{A}{x}+\frac{B}{x+5}\right)dx$

$x-5 = A(x+5)+Bx$

$x=-5 \quad -10 = -5B$
$2 = B$

$x = 0 \quad -5 = 5A \quad A = -1$

$= \int_{1}^{2}\left[\frac{-1}{x} + \frac{2}{x+5}\right]dx =$

$-\ln|x| + 2\ln|x+5|\Big|_{1}^{2} = -\ln 2 + 2\ln 7 - \left(-\cancel{\ln 1}^{\,0} + 2\ln 6\right)$

$= 2(\ln 7 - \ln 6) - \ln 2$

2b)

	u	dv
$+$	x^2-3x	e^{5x}
$-$	$2x-3$	$\frac{e^{5x}}{5}$
$+$	2	$\frac{e^{5x}}{25}$
	0	$\frac{e^{5x}}{125}$

$$y = \frac{(x^2-3x)e^{5x}}{5} - \frac{(2x-3)e^{5x}}{25} + \frac{2}{125}e^{5x} + C$$

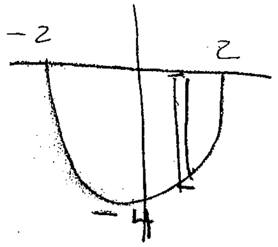
$$= \frac{e^{5x}}{5}\left(x^2-3x + \frac{3-2x}{5} + \frac{2}{25}\right) + C$$

2c) $\frac{1}{5}\int e^{5\sinh x}\, 5\cosh x \, dx$

$u = 5\sinh x$
$du = 5\cosh x \, dx$

$$\frac{1}{5}\int e^u \, du = \boxed{\frac{1}{5}e^{5\sinh x} + C}$$

3.



$\bar{x} = 0$ by symmetry

$\tilde{x} = x$

$\tilde{y} = \frac{0+(x^2-4)}{2}$

$\bar{y} = \frac{M_x}{m}$

$\boxed{(\bar{x},\bar{y}) = \left(0, \frac{-8}{5}\right)}$

$l = 0-(x^2-4) = 4-x^2$

$w = dx$

$dm = \delta(4-x^2)\,dx$

$$m = \delta\int_{-2}^{2}(4-x^2)\,dx = 2\delta\left[4x-\frac{x^3}{3}\right]_0^2 = 2\delta\left[8-\frac{8}{3}\right] = 2\delta\,\frac{16}{3} = \frac{32\delta}{3}$$

$$M_x = 2\delta\int_0^2 \frac{x^2-4}{2}(4-x^2)\,dx = \delta\int_0^2(-x^4+8x^2-16)\,dx$$

$$= \delta\left[\frac{x^5}{5} + \frac{8x^3}{3} - 16x\right]_0^2 = -\frac{32}{5} + \frac{64}{3} - \frac{32}{1} = 32\left(-\frac{1}{5}+\frac{2}{3}-1\right)$$

$\frac{-8(32)}{15}\delta$

4a) $L = \int_0^4 \sqrt{1+\sinh^2 x}\, dx = \int_0^4 \cosh x \, dx$

$= \sinh x \Big|_0^4 = \sinh 4 - \sinh 0 = \boxed{\sinh 4}$

b) $\frac{dy}{dx} + \frac{2}{x} y = \frac{\cos x}{x^2}$

$P(x) = \frac{2}{x}$ $\quad v = e^{\int \frac{2}{x} dx} = e^{\ln x^2} = x^2$

$Q(x) = \frac{\cos x}{x^2}$

$y = \frac{1}{x^2} \int \cancel{x^2} \frac{\cos x}{\cancel{x^2}} dx = \boxed{\frac{1}{x^2}(\sin x + C)}$