APPM 1360 MIDTERM #1 Summer 2008

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 4 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page.

1. (35 points) Consider the region enclosed by the functions $y = \sin(x) + 2$ and $y = \cos(x) + 2$, the y-axis and the line $x = 2\pi$.

   (a) Sketch these functions and shade the appropriate enclosed portion(s).

   (b) Write out the integral(s) that would determine the area of the region(s) between the two curves, the y-axis and the line $x = 2\pi$. Don't carry out the integration.

   ** For parts (c) - (e) consider just the region enclosed by the two functions for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$**

   (c) Suppose you were to create squares above the region with the base of the squares perpendicular to the x-axis. Write the integral that would determine the volume created by slicing. Don't carry out the integration.

   (d) Write out the integral that would determine the volume created by rotating the region about the x-axis using the disk/washer method. Don't carry out the integration.

   (e) Write out the integral that would determine the volume created by rotating the region about the y-axis using the shell method. Don't carry out the integration.

2. (25 points) Integrate the following:

   $$(a)\ \int_1^2 \frac{\sinh(\ln t)}{t}dt \qquad (b)\ \int \frac{dx}{\sqrt{16x^2-1}} \qquad (c)\ \int \frac{e^{\sqrt{t}}}{\sqrt{t}}dt \qquad (d)\ \int \sec(4\theta)d\theta$$

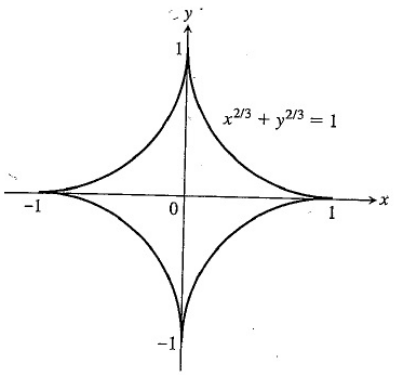
3. (15 points) Consider the region between the curve $y = 25 - x^2$ and the x-axis.

   (a) Draw the region.

   (b) Find the center of mass of the region if the density is constant.

4. (25 points)

   (a) Find the length of the astroid $x^{2/3} + y^{2/3} = 1$ pictured below. (Hint: solve for y, find the length of half the first quadrant piece $\frac{\sqrt{2}}{4} \leq x \leq 1$ and multiply by 8.)



   (b) Solve the differential equation: $x\frac{dy}{dx} + 2y = 1 - \frac{1}{x}$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name, and a grading key.

APPM 1360 — Formula Sheet —

A short table of integrals. In the following, $a \neq 0$.

(a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

(b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

(c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

(d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

(e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

(f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

(g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\operatorname{sech}^{-1}(u/a) + C$ for $0 < u < a$

(h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\operatorname{csch}^{-1}|u/a| + C$ for $u \neq 0$

(i) $\int \csc^2 u \, du = -\cot u + C$

(j) $\int \sec u \tan u \, du = \sec u + C$

(k) $\int \csc u \cot u \, du = -\csc u + C$

(l) $\int \tan u \, du = \ln|\cos u| + C = \ln|\sec u| + C$

(m) $\int \cot u \, du = \ln|\sin u| + C = -\ln|\csc u| + C$

(n) $\int \sec u \, du = \ln|\sec u + \tan u| + C$