APPM 1360 **MIDTERM #3** **Summer 2008**

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 6 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page.

1. (10 points) Answer the following questions with as much detail as possible. Justify any True/False questions with an explanation of why it's true or a counterexample to show it's false.
   (a) If the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converges and you estimate the series with the first five terms, then what is the greatest error in the estimation?
   (b) True or False: If the sequence a_n converges to zero, then the series $\sum_{n=1}^{\infty} a_n$ will converge.
   (c) True or False: If you show that $0 \le a_n \le b_n$ and the series $\sum_{n=1}^{\infty} b_n$ diverges, the the series $\sum_{n=1}^{\infty} a_n$ also diverges.
   (d) True or False: If series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then the series $\sum_{n=1}^{\infty} a_n b_n$ also converges.

2. (18 points) Use series to evaluate the following. Write your answer in closed form summation notation.

$$(a) \quad \lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos x} \qquad (b) \quad \int_0^1 x\sin(x^3)dx$$

3. (18 points) Do the following series converge or diverge? Be sure to name any test you use.

$$(a)\sum_{n=3}^{\infty} \frac{\ln n}{\ln\ln(n)} \qquad (b)\sum_{n=1}^{\infty} \tan\frac{1}{n} \qquad (c)\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

4. (18 points) Do the the following series converge or diverge? If the series converges, does it converge conditionally or absolutely? Be sure to name any test you use.

$$(a)\sum_{n=1}^{\infty} \frac{-1^n}{\sqrt{n}+\sqrt{n+1}} \qquad (b)\sum_{n=1}^{\infty}(-10)^{-n} \qquad (c)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$$

5. (18 points) For what values of x do the following series converge absolutely, converge conditionally or diverge? Be explicit about what x values do what. As usual, be sure to name any test you use.

$$(a)\sum_{n=1}^{\infty} \frac{x^n}{n^n} \qquad (b)\sum_{n=1}^{\infty} \frac{(x+4)^n}{n3^n} \qquad (c)\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$$

6. (18 points) Find the Taylor series generated by each of the given functions, centered at the given value of a.

$$(a) f(x) = e^{-1/x^3}, a = 0 \qquad (b) f(x) = 1/x^2, a = 1$$

Verify that the following information is clearly written on the front of your bluebook: your name and student ID number, your instructor's name, and a grading key.

APPM 1360 — Formula Sheet —

A short table of integrals. In the following, $a \neq 0$.

(a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

(b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

(c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

(d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

(e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

(f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

(g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

(h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$

(i) $\int \csc^2 u\, du = -\cot u + C$

(j) $\int \sec u \tan u\, du = \sec u + C$

(k) $\int \csc u \cot u\, du = -\csc u + C$

(l) $\int \tan u\, du = \ln|\cos u| + C = \ln|\sec u| + C$

(m) $\int \cot u\, du = \ln|\sin u| + C = -\ln|\csc u| + C$

(n) $\int \sec u\, du = \ln|\sec u + \tan u| + C$

Some useful limits (Table 8.1)

(a) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

(b) $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(c) $\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$

(d) $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$

(e) $\lim_{n\to\infty}(1 + \frac{x}{n})^n = e^x \quad$ (Any x)

(f) $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad$ (Any x)